Fading Modeling, MIMO Channel Generation, and Spectrum Detection for Wireless Communications

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Electrical Engineering

by

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To my parents
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PUBLICATIONS AND PRESENTATIONS


ABSTRACT OF THE DISSERTATION

Fading Modeling, MIMO Channel Generation, and Spectrum Detection for Wireless Communications

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We investigate the fading channel modelling, MIMO channel generation, and spectrum detection for wireless communications. We propose the modified hidden semi-Markov model (MHSMM) for modeling the flat fading envelope process. The MHSMM incorporate the time-variant statistics of the envelope process in a single model, which facilitates computations of the envelope probability density function and the autocorrelation function. A parameter estimation scheme is proposed. We demonstrate this parameter estimation scheme by simulated and experimental data, which are used in the IEEE 802.11 and the Global System for Mobile communication system.
Diversity techniques for various communication and MIMO systems exploit the spatial and temporal diversity attributes to mitigate the ill effects of the fading channels. We propose a unified approach capable of generating correlated flat-fading envelope processes with the desired auto-correlation functions, cross-correlation functions, and probability density functions (pdfs). The proposed approach utilizes the Gaussian vector autoregressive process and the inverse transform sampling techniques. Comparing to the past research focusing on generating fading channels of the same family, the novelty of the proposed approach is its capability to generate fading processes of heterogeneous pdfs. Examples including Nakagami, Rician, and Rayleigh channels are demonstrated.

Sensor networks have been shown to be useful in diverse applications. One of the important applications is the collaborative detection based on multiple sensors to increase the detection performance. To exploit the spectrum vacancies in cognitive radios, we consider the collaborative spectrum sensing by sensor networks in the likelihood ratio test (LRT) frameworks. We provide explicit algorithms to solve the LRT fusion rules, the probability of false alarm, and the probability of detection for the fusion center. We investigate the single-sensor detection and collaborative detections of multiple sensors under various fading channels, and derive testing statistics of the LRT with known fading statistics.
Chapter 1. Introduction

In wireless communication systems, the signals traverse the fading channels between the transmitter and the receiver. These fading channels degrade the performance of the wireless communications. The applications of the fading channel model include performance analyses, such as the bit error rate, the fade margin, the fade duration, the channel capacity, and the outage probability. Besides, certain power control, channel coding, and adaptive modulation schemes are built on the bases of fading channel models. To enable these analyses and applications, an accurate fading channel model plays a crucial role. The probability density functions (pdf) of the fading envelopes, i.e., the amplitude of the complex envelopes of the channel, have been investigated for many propagation scenarios. The Rayleigh distribution is the most commonly encountered pdf for the fading envelopes. The Rayleigh distribution is derived in the scenario where the scattered components are approximately equally strong. If there is a dominant component in the scattered components, e.g., the line-of-sight component, the Rician distribution is obtained. The multi-input multi-output (MIMO) is widely studied to mitigate fading channels and increase the capacity. The physical environments of the MIMO systems influence the auto-correlations and cross-correlations of the MIMO channels. With the increasing interests in MIMO systems, we explore the properties of the heterogeneous MIMO channels. In the cognitive radio system, the secondary users must seek spectrum hole, i.e. the spectrum not occupied, by using spectrum sensing techniques. The accuracies of the spectrum sensing techniques play the crucial roles in the cognitive radio. The accurate spectrum sensing facilitates the efficient utilization of the spectrum hole by
the secondary users, and reduces the potential interferences and channel conflicts to the primary users. We study the spectrum sensing problem using sensor networks in fading channels.

Figure 1.1. Fading effects.

Figure 1.2. Rayleigh channel.
1.1. Outline and Contributions

In Chapter 2, we investigate modified hidden semi-Markov model (MHSMM) for modeling the flat fading channels. In wireless communication systems, the physical channels degrade the signals in various ways, which are characterized by the fading channels. The major fading effects include reflections, diffractions, scattering, and shadowing. Fading phenomena impact the performance of wireless communication systems. We propose the modified hidden semi-Markov model (MHSMM) for modeling the flat fading envelope process. The properties of the envelope process are dominated by the physical fading processes and speeds of the mobile terminal. Thus, the statistics of the fading process may be non-stationary, due to different fading conditions over some time durations. The MHSMM incorporate these time-variant statistics of the envelope process in a single model, which facilitates computations of the envelope probability density function and the autocorrelation function. We provide a parameter estimation scheme to estimate the MHSMM parameters. We demonstrate the proposed parameter estimation scheme by simulated and experimental data. The simulation is based on the Global System for Mobile communication system, with scenarios chosen to mimic fading
conditions often encountered in mobile communications. The experiments were conducted in two scenarios, including cooperating and non-cooperating channel disturbances. Based on these data, the MHSMM is compared with the amplitude-based finite-state Markov chains model and the hidden Markov model. Results based on simulation and experimental data verify the advantages of the MHSMM and the effectiveness of the associated parameter estimation scheme.

In Chapter 3, we investigate the techniques to generate correlated MIMO channels. Diversity techniques for various communication and MIMO systems exploit the spatial and temporal diversity attributes to mitigate the ill effects of the fading channels. To evaluate these techniques, a method to generate multiple correlated fading channels is crucial. We propose a unified approach capable of generating correlated flat-fading envelope processes with the desired auto-correlation functions, cross-correlation functions, and probability density functions (pdfs). The proposed approach utilizes the Gaussian vector autoregressive process and the inverse transform sampling techniques. Comparing to the past research focusing on generating fading channels of the same family, the novelty of the proposed approach is its capability to generate fading processes of heterogeneous pdfs. Three examples are demonstrated. In the first example, the auto-correlated Nakagami channel is generated. The second example is designed to generate correlated 2x2 MIMO Rayleigh channels. In the third example, the proposed approach generates three correlated channels having Nakagami, Rician, and Rayleigh pdfs. The settings of the first two examples are adopted from previously published results, which are intended to verify the effectiveness of our proposed approach to tackle previously
known problems. The third example demonstrates the novelty of the proposed approach to generate correlated channels of heterogeneous pdfs.

In Chapter 4, we investigate the spectrum sensing in sensor networks. Sensor networks have been shown to be useful in diverse applications. One of the important applications is the collaborative detection based on multiple sensors to increase the detection performance. To exploit the spectrum holes in the cognitive radios, we consider the collaborative spectrum sensing by sensor networks in the likelihood ratio test (LRT) frameworks. In the LRT, the sensors make individual decisions. These individual decisions are then transmitted to the fusion center to make the final decision, which provides better detection accuracy than the individual sensor decisions. We provide the lowered-bounded probability of detection (LBPD) criterion as an alternative criterion to the conventional Neyman-Pearson (NP) criterion. In the LBPD criterion, the detector pursues the minimization of the probability of false alarm while maintaining the probability of detection above the pre-defined value. In cognitive radios, the LBPD criterion limits the probabilities of channel conflicts to the primary users. Under the NP and LBPD criteria, we provide explicit algorithms to solve the LRT fusion rules, the probability of false alarm, and the probability of detection for the fusion center. The fusion rules generated by the algorithms are optimal under the specified criteria. In the spectrum sensing, the fading channels influence the detection accuracies. We investigate the single-sensor detection and collaborative detections of multiple sensors under various fading channels, and derive testing statistics of the LRT with known fading statistics.
In Chapter 5, we conclude this research by summarizing our contributions in fading channel modeling, MIMO channel generation, and cooperative spectrum sensing.
Chapter 2. Modified Hidden Semi-Markov Model for Modelling the Flat Fading Channel

2.1. Introduction

In a flat fading channel, the channel gain imposes a multiplicative effect on the transmitted signal [2.2][2.4]. When the transmitting signal is narrowband with a constant envelope, the channel gain is proportional to the received signal envelope at the receiver. Thus, we use the terms, “envelope” and “channel gain,” interchangeably. Because the envelope determines the receiving signal strength, it is an important performance parameter in wireless communications. We focus on modeling statistical properties of the flat fading envelope.

One area of modeling the fading statistical properties is to model the probability density function (pdf) of the envelope. Various fading pdf models, such as the Rayleigh, Rician, and Nakagami distributions [2.1]-[2.4] have been investigated and accepted under different assumptions on the fading environments. For example, the Rician distribution is obtained by assuming a large number of weakly scattered signals and one dominant impinging signal, with its envelope larger than envelopes of the scattered signals [2.1].

To accurately characterize the envelope by the pdf, the physical fading conditions must persist consistently over sufficient time duration. In other words, a pdf model implicitly assumes the stationary properties of the envelope over the interested time duration. However, in real communication scenarios, the physical fading conditions may not always be stationary. For example, the dominant component in the Rician model may be shadowed due to the movement of the mobile user. Thus, the Rice k-factor [2.13] is
decreased. Changes in the physical fading conditions cause parameter changes in the envelope pdfs. Other more drastic changes in physical fading conditions may even create a completely different pdf for of the envelope. The non-stationary properties of the fading process also show up in ray-tracing techniques [2.25].

Besides the pdf property of the envelope, other characterizations of the random envelope process may be of interest. In the amplitude-based finite-state Markov chains model (AFSMCM, also abbreviated as AFSMC in [2.6]), the amplitude of the envelope is separated into non-overlapping intervals. The states of the AFSMCM represent the amplitude intervals of the envelope. By making state transitions as the time evolves, the envelope process is viewed as the outputs generated by the state transitions in the AFSMCM. In the AFSMCM, its acf is inherently specified by the state output and transition probability matrix [2.6]. Although the first order Markov property of the AFSMCM is supported by the analyses of mutual information [2.5], the AFSMCM has been shown to be not accurate in characterizing acfs [2.6]. In the fading model based on the hidden Markov model (HMM) [2.7][2.23], the state duration pdfs are inherently limited to geometric distributions of the self-transition probabilities of the states.

To remove some of the above-mentioned limitations, we propose a new modified hidden semi-Markov model (MHSMM) for modeling the flat fading envelope process. The proposed MHSMM is aimed to characterize the pdfs of the envelope, as well as the time-variant statistics in the envelope process. In order to make this model useful, we also propose explicit parameter estimation schemes for the proposed MHSMM. Various simulations and experimental data were collected to verify the usefulness of the
MHSMM. The associated accuracies are compared among the MHSMM, the AFSMCM, and the HMM. For the clear presentations, the notations defined in this chapter are self-contained and infer to this chapter only. Notations outside of this chapter are defined in the respective chapters.

2.2. MHSMM for NonStationary Fading Process

2.2.1. Physical Interpretations of Fading Channel in MHSMM

Since the physical process may not necessarily be stationary, these time-varying physical fading conditions cause the statistics of the envelope process to be time-varying. In order to account for these non-stationary properties, we view the envelope process as concatenations of piece-wise time-invariant stationary processes. Inside each state of the MHSMM, a stationary process is embedded. The embedded process in each state of the MHSMM represents outputs of its corresponding stationary duration of the envelope process. The rationale of using the MHSMM to characterize the fading channel is to use one state of the MHSMM to account for the stationary duration of the envelope process, which corresponds to the duration of the physical fading process remaining in the same condition, e.g., the possible high speed or low speed mobile movements, and the LOS or NLOS conditions. In certain models [2.3], the fading conditions can be separated into slow fading and fast fading scenarios, modeled separately. Corresponding to these interpretations, the MHSMM can be regarded as a model that incorporates both the slow fading and the fast fading conditions.

In general, the states of the MHSMM are recurrent, which corresponds to the fact that the physical fading scenarios can be revisited. The revisits of the same fading scenarios
cause the fading process to have approximately similar channel statistics when the transmitter and receiver are in similar physical conditions. These phenomena are observed in our experiments and the experiments of [2.26].

Our proposed MHSMM can be viewed as the regular hidden semi-Markov model (HSMM) [2.7] with the HSMM’s conditional state output probabilities replaced by the statistics of the individual random processes, specified by the conditional state pdfs and the acfs. Due to this modification of the regular HSMM, our proposed model is named as the modified hidden semi-Markov model (MHSMM). However, each state of the MHSMM corresponds to a time segment of the process, with each segment modeled as a stationary process having its own statistics, including a mean, an acf, an envelope pdf, and a state duration pdf.

2.3. MHSMM Definitions and Notations

The MHSMM consists of many states with an individual stationary process representing each state. Embedded inside each state is a random process with its own statistics. Transitions among the states are specified by the transition probability matrix. An example of four-state MHSMM is illustrated in Fig. 2.1. The notations are explained as follows:

1) The number of states is denoted as \( M \).

2) The \( M \) states of the MHSMM are denoted by \( \{S_1, S_2, \ldots, S_M\} \).

3) In the MHSMM, the time duration from the starting of one state to the end of the state is defined as one epoch. The state in the \( i \)-th epoch is defined as \( q_i \). The duration of
the $i$-th epoch is denoted as $\tau_i$. A realization of an example is illustrated in Fig. 2.2, where the epoch durations are $\tau_1 = 3$, $\tau_2 = 4$, and $\tau_3 = 6$. The realized states of the epochs are $q_1 = S_2$, $q_2 = S_1$, and $q_3 = S_4$.

2) The $M \times M$ state transition matrix is denoted by $A$. The elements in $A$ are denoted as $a_{ij}$, where $a_{ij} = P(q_{t+1} = S_j \mid q_t = S_i)$ for any $i, j$, $1 \leq i, j \leq M$, and $I$ is the index of any valid epochs. Since the state duration is explicitly specified in the MHSMM, without any loss of generality, we assume the state self-transition probability to be zero, i.e., $a_{ii} = 0$ for any $i$, $1 \leq i \leq M$.

3) The $1 \times M$ steady state probability vector of the $\{S_1, S_2, \ldots, S_M\}$ states is denoted by $\pi = [\pi_{S_1}, \pi_{S_2}, \ldots, \pi_{S_M}]$, where $\pi_i$ is the steady-state probability of $S_i$ for any valid $i$.

4) The state duration pdfs corresponding to $\{S_1, S_2, \ldots, S_M\}$ are individually denoted as $\{ P_{\text{dur}, S_1}(t), P_{\text{dur}, S_2}(t), \ldots, P_{\text{dur}, S_M}(t) \}$. For any $i$, $1 \leq i \leq M$, the $P_{\text{dur}, S_i}(t)$ can be interpreted as the pdf of an epoch length conditioned on the state $S_i$.

5) The acfs corresponding to the states of $\{S_1, S_2, \ldots, S_M\}$ are individually denoted by $\{ R_{S_1}(t), R_{S_2}(t), \ldots, R_{S_M}(t) \}$.

6) We denote the random variables (rvs) representing channel gains, observed at discrete time instants $\{t_1, t_2, \ldots, t_n\}$, by $\{X_1, X_2, X_3, \ldots, X_n\}$. The conditional envelope marginal pdfs conditioned on the states $\{S_1, S_2, \ldots, S_M\}$ are denoted by $\{ P_{S_1}(x), P_{S_2}(x), \ldots, P_{S_M}(x) \}$ individually. For the purpose of modeling the channel gains represented by the MHSMM, those $\{X_1, X_2, X_3, \ldots, X_n\}$ can be seen as the rvs.
representing the observable outputs of the MHSMM. The realization of these rvs are denoted by \( \{ x_1, x_2, x_3, \ldots, x_n \} \), as shown in Fig. 2.2.

![Figure 2.1. A 4-state example of the MHSMM.](image)

![Figure 2.2. An example on the realization of the MHSMM.](image)

Based on the above notations, the following quantities can be derived.

1) For any valid state, the steady-state percentage of time in the state \( S_i \) is given by

\[
\frac{\pi_i \mu_{dur(S_i)}}{\sum \pi_j \mu_{dur(S_j)}}. \tag{2.1}
\]

2) The overall envelope marginal pdf can be computed by
\[ P_s(x) = \frac{\pi_{S_t} \mu_{\text{dur}(S_t)}}{\sum \pi_{S_t} \mu_{\text{dur}(S_t)}} P_{x_1}(x) + \frac{\pi_{S_2} \mu_{\text{dur}(S_2)}}{\sum \pi_{S_t} \mu_{\text{dur}(S_t)}} P_{x_2}(x) + \ldots + \frac{\pi_{S_M} \mu_{\text{dur}(S_M)}}{\sum \pi_{S_t} \mu_{\text{dur}(S_t)}} P_{x_M}(x). \]  

(2.2)

In the example of Fig. 2.1, the state transitions are governed by the state transition probability \( a_{i,j} \) for any \( i, j \), \( 1 \leq i, j \leq 4 \). Inside each state, the state duration pdf \( P_{\text{dur},S_i}(\cdot) \) governs the state duration. The conditional envelope pdf \( P_{S_i}(\cdot) \) and the acf \( R_{S_i}(\cdot) \) govern the outputs of that state. In the realizations of Fig. 2.2, the outputs, \( \{x_1, x_2, x_3\} \), are generated from \( S_2 \), based on the conditional envelope pdf \( P_{S_2}(\cdot) \) and the acf \( R_{S_2}(\cdot) \). The duration \( \tau_1 \) is governed by the duration pdf \( P_{\text{dur},S_2}(\cdot) \). The same rule applies to subsequent outputs and states.

2.4. Parameter Estimation of MHSMM

When applying the above model to practical applications, we need to estimate the parameters of the MHSMM from a given sequence of channel realizations. The proposed scheme includes two steps: the sequence segmentation step and the state parameter estimation step. In the sequence segmentation step, we separate the observed channel sequence into segments corresponding to individual states of the MHSMM. Since these segments, represented by the state processes of the MHSMM, must be approximately stationary within individual states, the statistics are approximately equal within individual segments. On the contrary, the statistics are different between the segments represented by different states. The sequence segment step is designed to detect segment boundaries by exploiting the changes of the statistics, e.g., the mean and the acf of the sequences. After obtaining the segments, we characterize each segment by its mean value and
entropy value of the spectrum. We perform clustering algorithms to classify feature
gerpresentations of the segments into clusters. Each segment of those clusters individually
corresponds to a state of the MHSMM. In the state parameter estimation step, we use
these segments, obtained from the sequence segmentation step, to estimate the state
parameters of the states of the MHSMM.

2.5. Sequence Segmentation Step
In this step, we separate the observed channel sequence into segments corresponding to
individual states of the MHSMM, by detecting the change of the mean and acf values.
The segments are then clustered to form states of the MHSMM. The cluster algorithm is
based on similarities in the pairs of the mean and the acf values of the segments. By
denoting the observed realizations of \( \{X_1, X_2, X_3, \ldots, X_n\} \) as \( \{x_1, x_2, x_3, \ldots, x_n\} \), the
sequence segmentation step can be described as follows:

1) Compute the means of the observed channel gains based on a sliding window. This
operation generates local means, \( \{\mu_1, \mu_2, \mu_3, \ldots, \mu_n\} \), corresponding to the time instants
\( \{t_1, t_2, t_3, \ldots, t_n\} \).

2) Compute the spectrogram of the observed channel gains. This step generates the
local spectra, \( \{X_{t_1}(f), X_{t_2}(f), X_{t_3}(f), \ldots, X_{t_n}(f)\} \), corresponding to the time instants
\( \{t_1, t_2, t_3, \ldots, t_n\} \). We use these local spectra to detect changes in the acfs. Thus, the
entropies of the local spectra serve as good indicators for changes in the acfs.

3) Segment the local means \( \{\mu_1, \mu_2, \mu_3, \ldots, \mu_n\} \) by using the sliding window
segmentation approach [2.19]. Our purpose is to determine the time instants where the
\{\mu_1, \mu_2, \mu_3, \ldots, \mu_n\} show changes. Each segment represents a process with its own approximately time-invariant mean. In other words, the segment boundaries of \{\mu_1, \mu_2, \mu_3, \ldots, \mu_n\} separate \{x_1, x_2, x_3, \ldots, x_n\} into segments with approximately stationary means within individual segments. The time instants of the segment boundaries, determined by segmenting \{\mu_1, \mu_2, \mu_3, \ldots, \mu_n\}, are denoted as \{T_\mu, 1, T_\mu, 2, T_\mu, 3, \ldots, T_\mu, k_\mu\}.

4) Compute the sequence of the local entropies by computing the entropies of the absolute value of the local spectra, i.e., the entropies of the energy distributions of \|X_1(f)\|, \|X_2(f)\|, \|X_3(f)\|, \ldots, \|X_n(f)\|. The sequence of the entropies is denoted by \{e_1, e_2, e_3, \ldots, e_n\}.

5) Segment the \{e_1, e_2, e_3, \ldots, e_n\} by the sliding window segmentation approach [2.19]. The purpose of the operation is to determine the time instants where the \{e_1, e_2, e_3, \ldots, e_n\} show changes. Each segment represents a process with its own approximately time-invariant entropy. In other words, the segment boundaries of \{e_1, e_2, e_3, \ldots, e_n\} separate \{x_1, x_2, x_3, \ldots, x_n\} into segments with approximately stationary entropies within the individual segments. The time instants of the segment boundaries, determined by segmenting \{e_1, e_2, e_3, \ldots, e_n\}, are denoted by \{T_{e, 1}, T_{e, 2}, T_{e, 3}, \ldots, T_{e, k_e}\}.

6) Obtain the candidates of overall segment boundaries by sorting the union of the \{T_\mu, 1, T_\mu, 2, T_\mu, 3, \ldots, T_\mu, k_\mu\} and \{T_{e, 1}, T_{e, 2}, T_{e, 3}, \ldots, T_{e, k_e}\}. In other words, the obtained candidates of segment boundaries \{T_1, T_2, T_3, \ldots, T_{(k_\mu+k_e)}\} are equivalent to the sorted
We obtain and denote the segments as 
\[ \{ \bar{x}_1, x_2, x_3, \ldots, x_{(k_\mu+k_\nu+1)} \} \], where \( \bar{x}_1 = \{ x_1, x_2, \ldots, x_{T_1} \} \), \( \bar{x}_2 = \{ x_{T_1+1}, x_{T_1+2}, \ldots, x_{T_2} \} \), …, and 
\( \bar{x}_{(k_\mu+k_\nu+1)} = \{ x_{(k_\mu+k_\nu+1)+1}, x_{(k_\mu+k_\nu+1)+2}, \ldots, x_{N} \} \).

7) Compute the means and average entropies of the segments, \( \{ \bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_{(k_\mu+k_\nu+1)} \} \).
We denote the pairs of the means and the average entropies, corresponding to 
\( \{ \bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_{(k_\mu+k_\nu+1)} \} \), by \( \{ \mu_{\bar{x}_1}, e_{\bar{x}_1} \}, \{ \mu_{\bar{x}_2}, e_{\bar{x}_2} \}, \ldots, \{ \mu_{\bar{x}_{(k_\mu+k_\nu+1)}}, e_{\bar{x}_{(k_\mu+k_\nu+1)}} \} \).

8) By treating the mean and entropy pairs, \( \{ \mu_{\bar{x}_1}, e_{\bar{x}_1} \}, \{ \mu_{\bar{x}_2}, e_{\bar{x}_2} \}, \ldots, \{ \mu_{\bar{x}_{(k_\mu+k_\nu+1)}}, e_{\bar{x}_{(k_\mu+k_\nu+1)}} \} \) as points in the \( R^2 \), we use the \( \{ \mu_{\bar{x}_1}, e_{\bar{x}_1} \}, \{ \mu_{\bar{x}_2}, e_{\bar{x}_2} \}, \ldots, \{ \mu_{\bar{x}_{(k_\mu+k_\nu+1)}}, e_{\bar{x}_{(k_\mu+k_\nu+1)}} \} \), as feature representations of \( \{ \bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_{(k_\mu+k_\nu+1)} \} \) in the \( R^2 \) plane. Afterwards, we perform k-means clustering [2.20] on the mean and entropy pairs. One example is shown in Fig. 2.4(a). The number of clusters can be determined by using the elbow criterion [2.15] [2.16], the Davies-Bouldin Index [2.18], or the Dunn Index [2.17] based on the cluster centers from the k-means clustering. The estimated number of clusters is the estimated number of states for the MHSMM, which we denote \( \hat{M} \). By denoting the clusters as the MHSMM states individually, the states, \( \{ S_i \mid 1 \leq i \leq \hat{M} \} \), are created. After k-means clustering, we assign the clusters to their corresponding states in our intended MHSMM, and then assign all the segments to their corresponding states in the MHSMM.

In this step, each segment is assigned its own corresponding state in the MHSMM. We
denote the estimated states of all the segments as \( \{q_1, q_2, q_3, \ldots, q_{(k^+ + 1)}\} \), which represent the estimated MHSMM states of \( \{x_1, x_2, x_3, \ldots, x_{(k^+ + 1)}\} \) correspondingly.

9) For those segments which are consecutive and in the same cluster, we consider them derived from the over-segmentations of the segmentation algorithm. These consecutive and same-state segments are in fact in the same segment. So we combine the consecutive and same-state segments into a single segment. By combining the those segments, we form the final estimated segments \( \{\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_k\} \) and the estimated state sequence \( \{q_1, q_2, q_3, \ldots, q_k\} \), representing the estimated states of the segments \( \{x_1, x_2, x_3, \ldots, x_k\} \) correspondingly. The durations of the segments \( \{\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_k\} \) are denoted as \( \{\bar{D}_1, \bar{D}_2, \bar{D}_3, \ldots, \bar{D}_k\} \). Those estimated segment information, i.e., \( \{\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_k\} \), \( \{q_1, q_2, q_3, \ldots, q_k\} \), and \( \{\bar{D}_1, \bar{D}_2, \bar{D}_3, \ldots, \bar{D}_k\} \), will be used to perform the state parameter estimation step in next section.

2.6. State Parameter Estimation Step

In this step, we use the estimated segment information, i.e., \( \{\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_k\} \), \( \{q_1, q_2, q_3, \ldots, q_k\} \), and \( \{\bar{D}_1, \bar{D}_2, \bar{D}_3, \ldots, \bar{D}_k\} \), to estimate the state parameters. The procedures are described as follows.

1) The elements of state transition matrix \( \hat{A} \) can be estimated by \( \hat{a}_{ij} = \frac{N(S_i \rightarrow S_j)}{N(S_j)} \), for \( 1 \leq i, j \leq \hat{M} \), where \( N(S_i \rightarrow S_j) \) denotes the number of state transitions, from \( S_i \) to \( S_j \) in
the estimated state sequence \( \{q_1, q_2, q_3, \ldots, q_k\} \). The \( N(S_i) \) value denotes the number of states of \( S_i \) in the estimated state sequence \( \{q_1, q_2, q_3, \ldots, q_k\} \). This estimation approach for the transition matrix \( \hat{A} \) is based on the maximum likelihood estimation method [2.27].

2) The \( 1 \times M \) steady state probability \( \hat{\pi} \) can be computed by solving the equation \( \hat{\pi} = \hat{\pi} \hat{A} \).

3) The estimated state duration pdfs \( \{\hat{P}_{dur,S_i}(t), \hat{P}_{dur,S_2}(t), \ldots, \hat{P}_{dur,S_M}(t)\} \) can be computed from \( \{D_1, D_2, D_3, \ldots, D_k\} \) and \( \{q_1, q_2, q_3, \ldots, q_k\} \). For example, those \( \{D_j | q_j = S_i\} \), where \( 1 \leq j \leq k \), are treated as realizations of \( P_{dur,S_i}(t) \). Thus, the \( P_{dur,S_i}(t) \), can be estimated by performing pdf estimation technique on those \( \{D_j | q_j = S_i\} \). The pdf estimation can be performed by using the parametric approaches [2.13][2.14] or non-parametric approaches [2.21][2.22], depending on whether there are prior knowledge about the family of pdfs.

4) The estimated duration mean, \( \hat{\mu}_{dur(S_i)} \), of state \( S_i \) can be computed by the mean of the estimated duration pdf in each state, i.e., \( \hat{\mu}_{dur(S_i)} = \int t \cdot \hat{P}_{dur,S_i}(t) dt \), for \( 1 \leq i \leq M \).

5) The estimated acf, \( \hat{R}_S(t) \), of the state \( S_i \) can be estimated by computing the time-averaged acfs of the \( \{x_j | q_j = S_i, 1 \leq j \leq k\} \).
6) The estimated marginal envelope pdf, \( \hat{P}_{S_i}(x) \), in the state \( S_i \), can be computed by treating \( \{ x_j \mid q_j = S_i, 1 \leq j \leq k \} \) as realizations of \( P_{S_i}(x) \). The pdf estimation method includes the parametric [2.13][2.14] or the non-parametric approaches [2.21][2.22].

7) The steady-state percentage of time in the state \( S_i \) can be computed by using (2.1). The estimated overall envelope marginal pdf can be computed by using (2.2).

2.7. Implementing the AFSMCM and the HMM for comparisons

In order to evaluate the performance of the proposed MHSMM, we compare its performance to those based on alternative approaches of the AFSMCM and the HMM. The methodologies of estimating the parameters of the AFSMCM and the HMM are briefly described as follows.

The AFSMCM is implemented to have 80 states. In other words, the range of the envelope is divided into 80 intervals. For an AFSMCM with state transition probability \( a_{ij} \), its ML estimate is given by \( \hat{a}_{ij} = N(S_i \rightarrow S_j) / N(S_i) \) [2.27], where \( N(S_i) \) is the number of \( S_i \) and \( N(S_i \rightarrow S_j) \) is the number of transitions from \( S_i \) to \( S_j \) in the observed training sequence. Denoting the state outputs as the column vector \( \Upsilon \) and the transpose as T, the acf of the AFSMCM is given by \( R(m) = \frac{1}{N} \Upsilon^T (P^m)^T \Upsilon \) [2.6].

For the HMM, the range of the envelope is also divided into 80 intervals. The centers of the 80 intervals are the outputs of the states in the HMM. The HMM is implemented to have 4 states. Each state has its own probability mass function (pmf) with the domain spanning over the centers of the 80 envelope intervals. The iterative Baum-Welch method...
with forward-backward variables [2.7] is implemented to estimate the parameters in the HMM. This approach is optimal in the likelihood sense, although only the local maximum likelihood estimation is guaranteed due to the many local maxima in the parameter space [2.7]. Then the acf of the HMM is computed by these HMM parameters, as described in [2.23].

In this chapter, the parameters of the AFSMCM and the HMM are all estimated using the above-mentioned approaches.

2.8. Simulation

The simulation is designed to characterize the GSM system [2.8]. The fading envelope process is simulated to be a Rician process, with its acf specified by the Clarke model. The Rician marginal envelope distribution is given by

$$f_{Ric}(x) = \frac{2(k_R + 1)x}{\Omega} \exp\left(-k\frac{(k_R + 1)x^2}{\Omega}\right)I_0\left(2\sqrt{\frac{k_R(k_R + 1)}{\Omega}}x\right),$$

(2.3)

where $x \geq 0, k_R \geq 0, \Omega \geq 0$. The Clarke model [2.24] theoretically derives the temporal acf as $E[x(t)x(t-\Delta t)] = J_0(2\pi f_D \Delta t)$, where $J_0(\cdot)$ is the 0-th order Bessel function of the first kind and the $f_D$ is the maximum Doppler frequency shift. For a transmitter with the carrier frequency $f_c$, the mobility speed $v$, and the speed of electromagnetic wave $c$, the maximum Doppler frequency shift is expressed as $f_D = v \cdot f_c / c$. In the Clarke model, the power spectrum of the envelope process can also be specified by

$$X(f) = \frac{1}{\sqrt{1-(f/f_D)^2}}, \quad |f| \leq f_D.$$
different states. The lognormal distribution is then used as the state duration pdf, expressed as

\[ f_{\text{lognormal}}(t; \mu_n, \sigma) = \frac{\exp(-\left(\ln(t) - \mu_n\right)^2/(2\sigma^2))}{t\sigma\sqrt{2\pi}}, \]  

(2.4)

where \(0 \leq t < \infty, -\infty \leq \mu_n \leq \infty, \sigma \geq 0\).

In the GSM system [2.8], the nominal carrier frequency is 900 MHz. The power level can be from 39 dbm to 5 dbm. Thus, a 10 db difference between the LOS and the NLOS scenario is assumed to be reasonable. Of course, a 10 db difference in power (watt) corresponds to a 20 db difference in envelope (voltage). There are four states in the simulation, described as follows:

**State 1**: The mobile is moving in a high speed at 30 m/s, in the NLOS scenario. The mean of the received signal envelope is -10 db, with respect to the reference level. Based on the 900 MHz of the GSM carrier frequency, the Doppler shift is \(f_D = 90 \text{ Hz}\). Since it is in the NLOS scenario, a reasonable value of \(k_R\) in the Rician process is taken to be 1, i.e., no dominant components in the scattered signals. The state duration is generated by the lognormal distribution with a mean equal to 10 seconds, with the parameters of \((\mu_n, \sigma) = (2, 0.7779)\) in (2.4).

**State 2**: The mobile is moving in a high speed at 30 m/s, in the LOS scenario. The mean of the received signal envelope is 0 db, i.e., equal to the reference level. The Doppler shift is \(f_D = 90 \text{ Hz}\). Because of the LOS scenario, the \(k_R\) in the Rician process is taken to be 10, i.e., the dominant component has the envelope 10 times larger than the envelopes.
of the scattered signals. The state duration is generated by the lognormal distribution with a mean equal to 30 seconds, with parameters of \((\mu_m, \sigma) = (2.5, 0.9957)\) in (2.4).

**State 3**: The mobile is moving in a low speed at 3 m/s, in the NLOS scenario. The mean of the received signal envelope is -10 db. The Doppler shift is \(f_D = 9\) Hz. Because of the NLOS scenario, the \(k_r\) in the Rician process is taken to be 1. The state duration is generated by the lognormal distribution with a mean equal to 50 seconds, with parameters of \((\mu_m, \sigma) = (3, 0.8958)\) in (2.4).

**State 4**: The mobile is moving in a low speed at 3 m/s, in the LOS scenario. The mean of the received signal envelope is 0 db. The Doppler shift is \(f_D = 9\) Hz. Because of the LOS scenario, the \(k_r\) in the Rician process is taken to be 10. The state duration is generated by the lognormal distribution with a mean equal to 70 seconds, with parameters of \((\mu_m, \sigma) = (3.5, 0.6146)\) in (2.4).

The state transition probability is taken as

\[
P = \begin{bmatrix}
0 & 0.2 & 0.2 & 0.6 \\
0.3 & 0 & 0.5 & 0.2 \\
0.3 & 0.3 & 0 & 0.4 \\
0.4 & 0.4 & 0.2 & 0
\end{bmatrix},
\]

which specifies the steady state probability to be

\[
\pi = [0.2529\quad 0.2335\quad 0.2251\quad 0.2885].
\]

In our simulation, the envelope process is generated by the above parameters inside **state 1 to 4**, with state transitions governed by \(P\). The initial state is generated by \(\pi\). The envelope process is simulated for 10437 seconds, where 400 states are realized. The envelope process is sampled at 1000 samples/second.
After obtaining the envelope sequence, we estimate the MHSMM parameters by the proposed sequence segmentation step and the state parameter estimation step. To estimate the $P_s(x)$, we use the parametric approach on the Rician distributions. For each state, we estimate its own envelope pdf parameters individually. The maximum likelihood estimator for $\Omega$ in (2.3) is given by $\hat{\Omega} = \frac{1}{n} \sum_{i=1}^{n} x_i$ [2.13]. For $k_r$ in (2.3), the moment method [2.13] is used by computing $k_r = \frac{\sqrt{1-\gamma}}{1-\sqrt{1-\gamma}}$ and $\gamma = \frac{Var[x^2]}{(E[x^2])^2}$. To estimate $P_{dur.s}(t)$, the parametric approach is used on the lognormal distribution [2.14]. For each state, the parameters of the state duration pdf are estimated individually. The parameters of (2.4) are estimated by $\mu = \ln(E[x]) - \frac{1}{2} \ln \left( 1 + \frac{Var[x]}{(E[x])^2} \right)$ and $\sigma^2 = \ln \left( 1 + \frac{Var[x]}{(E[x])^2} \right)$. Those $E[x]$, $Var[x]$, and $Var[x^2]$ are computed by their time-average counterparts.

The simulated envelope sequence for 0 to 400 seconds is shown in Fig. 2.3(a). The local mean and entropy values are shown in Fig. 2.3(b)(c). The details of the simulated sequence from 40 seconds to 60 seconds are shown in Fig. 2.3(d)(e)(f). The estimated transition points, computed by the proposed parameter estimation scheme, and the true transition points are also shown in Fig. 2.3. The estimated transition points are close to the true transition points.
Figure 2.3. Envelope realizations: (a) the realized envelopes from 0-400 seconds with the estimated and true transition points, (b) the local mean of the envelope realizations in (a), and (c) the local entropy of the envelope realizations in (a). Details of the 40-60 seconds are shown in (d)(e)(f).
In Fig. 2.4(a), the feature representations are visualized to have 4 clusters. In Fig. 2.4(b)-(d), the elbow rule, the Davis-Bouldin Index, and the Dunn Index consistently indicate 4 as the suggested number of cluster centers. Thus, the 4-state MHSMM was used to model this fading sequence. By performing the proposed parameter estimation scheme, the results are shown in Fig. 2.5. In Fig. 2.5(a), the theoretical pdf is computed by the parameters specified in the simulation. The estimated MHSMM pdf and the AFSMCM pdf are close to the theoretical pdf. The HMM pdf can be seen to obviously deviate from the theoretical pdf. In Fig. 2.5(b), the cdfs are compared. The results show the MHSMM cdf and the AFSMCM cdf are close to the theoretical cdf, while the HMM cdf deviates from the theoretical cdf. The D-statistics, from the Kolmogorov-Smirnov test (KS-test) with respect to the theoretical cdf, of the MHSMM cdf, the AFSMCM cdf, and the HMM cdf are 0.0219, 0.0637, and 0.1695, which show the MHSMM has the best-fitted pdf among the 3 models. These comparisons show the effectiveness of the proposed MHSMM and the associated parameter estimation scheme.
Figure 2.4. (a) Feature representations of the segments from the simulation. (b) The elbow phenomenon, (c) the Davis-Bouldin index, and (d) the Dunn index for determining the number of clusters.
Figure 2.5. (a) The theoretical pdf, the estimated MHSMM pdf, the estimated AFSMCM pdf, and the estimated HMM pdf. (b) The theoretical cdf, the estimated MHSMM cdf, the estimated AFSMCM cdf, and the estimated HMM cdf.

2.9. Experiments

Experiments have been conducted in two scenarios, i.e., the patio (called Experiment 1) and the hallway (called Experiment 2) of UCLA. In both experiments, the carrier frequency was set at 2.4 GHZ. The signal envelope was collected at 100 samples per second. The measurements of 3000 seconds were collected, individually from each experiment. The data of 3000 seconds, from each scenario, were separated into the first 1500 seconds, denoted as the data set 1, and the second 1500 seconds, denoted as the data set 2. Our purpose was to use the data set 1 to estimate the model parameters, and then use the data set 2 to study the accuracies. Before the experiments, background measurements were observed to ensure that no existing local services, e.g., IEEE 802.11 systems, were operating near the experimental sites.

2.9.1. Experiment 1-The Patio Scenario

Experiment 1 was conducted in the patio at the second floor of the UCLA Engineering IV building. The transmitter and the receiver were in the LOS positions. The patio is a
semi-open area, with reflectors and scatters surrounding the transmitter and the receiver. The dynamics of the fading channel were caused by a pre-arranged personnel walking through the LOS path of the channel. The person walked back and forth periodically at a regular walking speed. The experiment was conducted at the time when the site was quiescent without other non-cooperating dynamic fading disturbances, with the pre-arranged person being the only dynamic channel disturbance.

The results are shown in Fig. 2.6 and 2.7. The number of clusters is suggested to be two by the verification indexes of the Davis-Bouldin and the Dunn Index. Thus, the two-state MHSMM is employed to further estimate the state parameters. The data pdfs in Fig. 2.6(a) are constructed by the kernel density estimation (KDE) method [2.21][2.22]. The estimated MHSMM pdf is close to the data pdfs from both the data set 1 and 2, shown in Fig. 2.6(a). The estimated MHSMM cdf is also close to the data cdfs from both data set 1 and 2, shown in Fig. 2.6(b). Comparing empirical cdf of the data set 1 with the estimated cdfs of the models, the D-statistics from the KS-test are 0.2167 (MHSMM), 0.0771 (AFSMCM), and 0.3115 (HMM), which show the AFSMCM has the best-fitted pdf. Comparing empirical cdf of the data set 2 with the model cdfs estimated by the data set 1, the D-statistics of the KS-test are 0.1637 (MHSMM), 0.1396 (AFSMCM), and 0.2444 (HMM), which show the AFSMCM also has the best-fitted pdf. The estimated acfs for the two states of the MHSMM are shown in Fig. 2.7. The acfs of the AFSMCM and the HMM are also shown in Fig. 2.7. Although, in this case, the AFSMCM is slightly better in fitting the pdfs, it is worth noticing that the capability of fitting the pdfs is only one of the performance metrics for fading models. For characterizing the acfs, the MHSMM
shows more flexibility. When selecting models, the importance of the model accuracy needs to be considered based on specific applications.

Figure 2.6. (a) Pdfs from the experimental data and the estimated pdfs. (b) Cdfs from the experimental data and the estimated cdfs.

Figure 2.7. (a) Estimated acfs of the MHSMM state 1; (b) MHSMM state 2; (c) AFSMCM; (d) HMM.
2.9.2. Experiment 2-The Hallway Scenario

Experiment 2 was conducted in the hallway of the first floor of the UCLA Engineering IV building. The transmitter and the receiver were in the NLOS positions. The hallway is in an indoor environment with many reflectors and scatters. This experiment was conducted during rush hours, around 10 o’clock in the morning of a working week day. The dynamics of the fading channel were caused by many non-cooperating persons, walking through or waiting for the elevators. We did not intend to put any controlled dynamic fading disturbances. This setup was intended to capture the fading channel characteristics under normal busy hallway conditions.

These results are shown in Fig. 2.7. The number of clusters is suggested to be four by the Davis-Bouldin Index and the Dunn Index. Thus, the four-state MHSMM is employed to further estimate the state parameters. The estimated pdf of the MHSMM is close to the data pdfs, constructed by the KDE method, from both the data set 1 and 2, as shown in Fig. 2.8(a). The estimated cdfs of the MHSMM are also close to the data cdfs from both data set 1 and 2, as shown in Fig. 2.8(b). Comparing the empirical cdf of the data set 1 with the estimated model cdfs, the D-statistics of the KS-test are 0.0276 (MHSMM), 0.0435 (AFSMCM), and 0.1643 (HMM), which show the MHSMM is the best-fitted. Comparing the empirical cdf of the data set 2 with the model cdfs estimated by the data set 1, the D-statistics of the KS-test are 0.0699 (MHSMM), 0.0734 (AFSMCM), and 0.1521 (HMM), which again show the MHSMM is the best-fitted. The estimated acfs for the four states of the MHSMM are shown in Fig. 2.9. The acfs of the AFSMCM and the HMM are also shown in Fig. 2.9. The MHSMM is able to characterize different acfs in
different conditions. Summarizing the results from both the patio and the hallway experiments, we can conclude the capability of the MHSMM is comparable to that of the AFSMCM in characterizing the pdfs and cdfs, while the MHSMM is more flexible in characterizing the acfs.

Figure 2.8. (a) Pdfs from the experimental data and the MHSMM. (b) Cdfs from the experimental data and the MHSMM.
Figure 2.9. (a-d) Estimated acfs of the MHSMM state 1 to 4. (e) Estimated acfs of the AFSMCM. (f) Estimated acfs of HMM.
2.10. Discussion

In the HMM and the hidden semi-Markov model (HSMM), there exist iterative algorithms which adjust the model parameters to increase the likelihood [2.7], based on the iterative Baum-Welch method with the forward-backward variables. However, the iterative algorithms remain to be improved in the following aspects. First, we need to know the number of states, outputs of the states, etc., in order to apply the iterative algorithm. In real applications, we lack the required information about the number of states and outputs of the states, which can only be chosen heuristically. Besides, the iterative algorithms often achieve local maxima. The maximum likelihood estimation is difficult to achieve in practice. Secondly, the formulation of the iterative algorithm is mathematically tractable under the white process assumption, i.e., the outputs at different time instants are un-correlated. In the colored process, the iterative algorithms are difficult be formulated explicitly. In contrast to these limitations, the MHSMM is designed to provide the freedom in characterizing the acf. Besides, by using the parameter estimation scheme of the MHSMM, the difficulties in the initial parameter choices, the local ML, and the formulation of the colored process can be avoided.

When estimating the pdfs of \( P_{\text{dur},S_i}(t) \) and \( P_{S_i}(x) \), we can use parametric or non-parametric approaches. If the prior knowledge about our interested scenario is available, we may consider using the family of pdfs motivated by the prior knowledge. By using the prior-known family of pdfs, the pdf estimation problem is reduced to estimate the parameters of the family of pdfs. If there is no prior knowledge available, non-parametric approaches must be employed, e.g., the KDE approach [2.21][2.22].
2.11. References


Chapter 3. A Unified Approach for Generating Cross-correlated and Auto-correlated MIMO Fading Envelope Processes

3.1. Introduction

In this chapter, we focus on the frequency-flat fading channels [3.1]. The Jake’s model describes the auto-correlation function (ACF) of the fading process by the zeroth-order Bessel function of the first kind [3.2][3.3]. The ACFs and cross-correlation functions (CCFs) of multiple channels are studied under various propagation environments in [3.4][3.5]. Various transmission schemes under fading have been studied. For example, in [3.6][3.7], modulation schemes are studied in the Rayleigh and Nakagami fading channels. In [3.8], turbo codes are investigated in correlated Rayleigh fading channels. To verify these analyses by simulations, the generation of the correlated fading processes for analyses and simulations is crucial. The Rayleigh, Rician, Nakagami, and Weibull pdfs [3.10] are studied and applicable in certain propagation environments. In the multi-channel systems where each channel encounters different fading effects, the channels are possibly heterogeneous with different fading envelope pdfs.

In past research, various techniques exploiting the diversities have been investigated and proven to be useful. For example, the techniques of diversity combining [3.11][3.12] and the MIMO system [3.13] utilize the spatial diversity to improve performances. The performances of diversity receivers in correlated Weibull fading channels are investigated in [3.14]. The space-time codes [3.15] exploit the temporal and spatial diversities to improve performances. The performances of those techniques highly depend on the
spatial and temporal correlations. In the diversity combining and MIMO systems, the
temporal and spatial correlations of the multiple channels are described by the ACFs and
CCFs. In this chapter, the ACFs and the CCFs are collectively called correlations.
Generally, the multiple processes are characterized by the correlations. In the analysis,
simulation, and experimental measurements, the generation of correlated fading
processes facilitates the evaluations and verifications of the performances of the multi-
channel systems. Previous research has been focused on generating correlated multiple
fading envelope processes with pdfs of the same family, including the correlated
Rayleigh fading envelopes [3.16][3.17][3.18], the correlated Rician fading envelopes
[3.2], and the correlated Nakagami processes [3.19][3.20][3.21]. However, comparing to
the past research focusing on the pdfs of the same family, we notice the generation of
correlated fading envelope processes of pdfs from heterogeneous families may be needed
in some applications. For example, the multiple channels of heterogeneous families could
co-exist in the scenario where the signal transmission paths encounter heterogeneous
scatters, reflections, and diffractions. Therefore, our goal is to propose a unified approach
which has the capability to generate multiple envelope processes of different families. To
be more specific, for any given ACFs, CCFs, and marginal pdfs, our proposed approach
generates the fading envelope processes with the desired ACFs, CCFs, and the marginal
pdfs. The novelty of the proposed approach is to allow the pdfs of the processes to be
taken from different families. For the clear presentations, the notations defined in this
chapter are self-contained and infer to this chapter only. Notations outside of this chapter
are defined in the respective chapters.
3.2. The Proposed Approach
3.2.1. Concepts and Notations

The basis of our approach utilizes the Gaussian vector autoregressive (AR) process as the driving process. Conceptually, the samples generated by the Gaussian vector AR process are processed by the inverse transform sampling [3.22][3.23] to generate the fading envelope processes with the desired pdfs. The correlations of the fading envelope processes are determined by controlling the correlations in the Gaussian vector AR process.

Properties of Gaussian AR have been extensively investigated in [3.2][3.24][3.3]. One of their useful properties is the matrix-valued Yule-Walker equation [3.2][3.3], which yields the parameters of the Gaussian vector AR process for the given correlations. Because of these useful and well-formulated properties, the Gaussian vector AR process is selected as the driving process in our design. The block diagram of the proposed approach is illustrated in Fig. 3.1. The Gaussian vector AR process, shown at the left-hand-side of Fig. 3.1, is used to yield the fading processes. The samples generated by the Gaussian vector AR process are transformed by the inverse transform sampling, shown in the middle blocks of Fig. 3.1. The output of the inverse transform sampling generates the desired fading envelopes, shown at the right-hand-side of Fig. 3.1. For the Gaussian vector AR process, the value of the $i$-th process at time $n$ is denoted as $x_{i,n}$. For the fading envelope processes, the value of the $i$-th process at time $n$ is denoted as $y_{i,n}$, where $x_{i,n}$ and $y_{i,n}$ are real numbers. We denote normalized Gaussian( with zero mean and unit
variance) cumulative distribution function (cdf) as $\Phi(x_{i,n})$, the normalized Gaussian pdf as $N(x_{i,n})$, the cdf of the $i$-th envelope process as $F_i(y_i)$, and the pdf of the $i$-th process as $f_i(y_i)$. In the Gaussian vector AR process, the CCF of the $i$-th process with the $j$-th process is denoted by $r_{ij} = E[x_{i,n+\tau}x_{j,n}]$. The discussions in this chapter are based on the discrete time domain, i.e., all the time indexes are integers. The number of processes is denoted by $M$.

Figure 3.1. Proposed approach using the inverse transform sampling.

For the fading envelope processes, the normalized CCF of the $i$-th and the $j$-th envelope processes is denoted by $\rho_{ij,\tau} = \frac{1}{\sigma_{y_i,\sigma_{y_j}}}E[(y_{i,n+\tau} - \mu_{y_i})(y_{j,n} - \mu_{y_j})]$, where $\mu_{y_i}$ is the mean and $\sigma_{y_j}$ is the standard deviation of the $i$-th envelope process. Then $\rho_{ii,\tau}$ is the ACF of $i$-th envelope process with the property, $\rho_{ii,0} = 1$. It is noticed that, by re-formulating $\rho_{ij,\tau}$, we obtain un-normalized autocorrelations, $E[(y_{i,n} - \mu_{y_i})(y_{j,n} - \mu_{y_j})] = \sigma_{y_i}^2\rho_{ii,0} = \sigma_{y_j}^2$, 

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which need not be equal to 1. In other words, the processes discussed in this chapter are not restricted to be unit power. However, as opposed to the un-normalized ACFs and CCFs, the normalized ACFs and CCFs simplify the analyses and comparisons of the results. Therefore, the discussions focus on and refer to the normalized ACFs and CCFs unless stated otherwise.

Using the above notations, the proposed approach provides an algorithm that, by controlling \( r_{ij,\tau} \) of \( x_{i,n} \), generates the multiple fading envelope processes, \( y_{i,n} \), with the desired \( \rho_{ij,\tau} \) and \( f_j(y_i) \), for \( i=1,...,M \) and \( j=1,...,M \).

The Gaussian vector AR of \( M \) processes with the order \( p \) is represented by

\[
X_n = -\sum_{i=1}^{p} A_i X_{n-i} + W_n, \quad \text{where} \quad X_n = [x_{1,n}, x_{2,n}, ..., x_{M,n}]^T \quad \text{is the} \quad M \times 1 \quad \text{vector at time} \quad n, \quad \text{the} \quad A_i \quad \text{is the} \quad M \times M \quad \text{coefficient matrix, and the} \quad W_n = [w_{1,n}, w_{2,n}, ..., w_{M,n}]^T \quad \text{is the} \quad M \times 1 \quad \text{Gaussian vector.} \quad \text{In the Gaussian vector AR model, the} \quad W_n \quad \text{is a temporally white zero-mean Gaussian vector process, i.e.,} \quad E[W_n] = 0_{M \times 1} \quad \text{and} \quad E[W_n W_n^T] = 0_{M \times M} \quad \text{for any} \quad i \neq 0. \quad \text{The correlation matrix of} \quad W_n \quad \text{at time lag zero is denoted as} \quad Q, \quad \text{i.e.,} \quad E[W_n W_n^T] = Q. \quad \text{The matrices representing the correlation structure of the Gaussian vector AR are denoted by the} \quad M \times M \quad \text{matrix}
\]

\[
R_{xx,\tau} = E[X_{n+\tau} X_n^T] = \begin{bmatrix}
   r_{1,1} & \cdots & r_{1,M,\tau} \\
   \vdots & \ddots & \vdots \\
   r_{M,1,\tau} & \cdots & r_{M,M,\tau}
\end{bmatrix}, \quad (3.1)
\]

the \( Mp \times Mp \) matrix
\[
R_{xx} = \begin{bmatrix}
R_{xx,0} & \cdots & R_{xx,-p+1} \\
\vdots & \ddots & \vdots \\
R_{xx,p-1} & \cdots & R_{xx,0}
\end{bmatrix},
\]  

(3.2)

and the \(Mp \times M\) matrix

\[
V = -E\begin{bmatrix}
[X_{n+1}X_n^T] \\
[X_{n+2}X_n^T] \\
\vdots \\
[X_{n+p}X_n^T]
\end{bmatrix} = \begin{bmatrix}
R_{xx,1} \\
R_{xx,2} \\
\vdots \\
R_{xx,p}
\end{bmatrix}.
\]  

(3.3)

The coefficient matrix is denoted by the \(Mp \times M\) matrix

\[
A = \begin{bmatrix}
A_1 & A_2 & \cdots & A_p
\end{bmatrix}^T.
\]  

(3.4)

The matrix-valued Yule-Walker equation is expressed as

\[
R_{xx}A = V.
\]  

(3.5)

The \(M \times M\) correlation matrix, \(Q\), of the noise in the vector AR can be obtained from

\[
Q = R_{xx,0} + \sum_{k=1}^{p} R_{xx,-k}A_k^T.
\]  

(3.6)

Thus, by (3.5) and (3.6), the matrix-valued Yule-Walker equation yields the parameters of the Gaussian vector AR process, i.e., \(A_i\) and \(Q\), for the specified correlation coefficients, \(r_{ij,\tau}\). Since we can solve \(A_i\) and \(Q\) from the \(r_{ij,\tau}\), the next required step is to solve for \(r_{ij,\tau}\) from the desired \(\rho_{ij,\tau}\) and \(f_i(y_i)\). In other words, based on this framework, we need to establish the relationships among \(\rho_{ij,\tau}\), \(f_i(y_i)\), and \(r_{ij,\tau}\), such that the \(r_{ij,\tau}\) can be computed from the \(\rho_{ij,\tau}\) and \(f_i(y_i)\). Relationships among \(r_{ij,\tau}\), \(\rho_{ij,\tau}\), and \(f_i(.)\) are given in the next section.

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3.2.2. Deriving the Equations of $\rho_{ij}, f_i(y_i),$ and $r_{ij}$

Based on the framework in Fig. 3.1, we have $y_{i,n} = F_i^{-1}(\Phi(x_{i,n})),$ where $x_{i,n}$ represents the sample of the $i$-th process of the Gaussian vector AR process at time $n.$ The joint pdf of the samples of the $i$-th process at time $n + \tau,$ $x_{i,n+\tau},$ and the $j$-th process at time $n,$ $x_{j,n},$ is expressed by

$$f_{i,j}(x_{i,n+\tau}, x_{j,n}) = \frac{1}{2\pi \left| \begin{bmatrix} 1 & r_{ij} \\ r_{ij} & 1 \end{bmatrix} \right|^{1/2}} \exp \left( -\frac{1}{2} \begin{bmatrix} x_{i,n+\tau} \\ x_{j,n} \end{bmatrix}^T \begin{bmatrix} 1 & r_{ij} \\ r_{ij} & 1 \end{bmatrix}^{-1} \begin{bmatrix} x_{i,n+\tau} \\ x_{j,n} \end{bmatrix} \right).$$  \hspace{1cm} (3.7)

Substituting $y_{i,n+\tau} = F_i^{-1}(\Phi(x_{i,n+\tau}))$ and $y_{j,n+\tau} = F_j^{-1}(\Phi(x_{j,n+\tau}))$ in (3.7), we obtain

$$f_{i,j}(x_{i,n+\tau}, x_{j,n}) \bigg|_{y_{i,n+\tau} = F_i^{-1}(\Phi(x_{i,n+\tau}))} = \frac{1}{2\pi \left| \begin{bmatrix} 1 & r_{ij} \\ r_{ij} & 1 \end{bmatrix} \right|^{1/2}} \exp \left( -\frac{1}{2} \begin{bmatrix} \Phi^{-1}(F_i(y_{i,n+\tau})) \\ \Phi^{-1}(F_j(y_{j,n})) \end{bmatrix}^T \begin{bmatrix} 1 & r_{ij} \\ r_{ij} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \Phi^{-1}(F_i(y_{i,n+\tau})) \\ \Phi^{-1}(F_j(y_{j,n})) \end{bmatrix} \right).$$ \hspace{1cm} (3.8)

Denoting $Y = [y_{i,n+\tau} \ y_{j,n}]^T$ and $X = [x_{i,n+\tau} \ x_{j,n}]^T,$ by Jacobian transformation, we obtain

$$f_{i,j}(y_{i,n+\tau}, y_{j,n}) = \frac{f_{i,j}(x_{i,n+\tau}, x_{j,n})}{\left| J(Y, X) \right|_{y_{i,n+\tau} = F_i^{-1}(\Phi(x_{i,n+\tau}))}^{\text{evaluated at } x_{j,n} = F_j^{-1}(\Phi(y_{j,n}))}}.$$ \hspace{1cm} (3.9)

where the Jacobian $\left| J(Y, X) \right|$ is expressed as
\[
\begin{align*}
|J(Y, X)|_{x_{i,n+\tau} = y_{i,n+\tau}}^{x_{i,n} = y_{i,n}} &= \begin{vmatrix}
\frac{\partial y_{i,n+\tau}}{\partial x_{i,n+\tau}} & \frac{\partial y_{i,n+\tau}}{\partial y_{j,n}} & \frac{\partial y_{i,n+\tau}}{\partial x_{j,n}} \\
\frac{\partial y_{j,n}}{\partial x_{i,n+\tau}} & \frac{\partial y_{j,n}}{\partial y_{j,n}} & \frac{\partial y_{j,n}}{\partial x_{j,n}} \\
\frac{\partial x_{i,n+\tau}}{\partial x_{i,n+\tau}} & \frac{\partial x_{i,n+\tau}}{\partial y_{j,n}} & \frac{\partial x_{i,n+\tau}}{\partial x_{j,n}} \\
\end{vmatrix}
= \begin{vmatrix}
\frac{\partial F^{-1}_i(\Phi(x_{i,n+\tau}))}{\partial x_{i,n+\tau}} & 0 & 0 \\
\frac{\partial F^{-1}_j(\Phi(x_{j,n}))}{\partial y_{j,n}} & \frac{\partial F^{-1}_j(\Phi(x_{j,n}))}{\partial y_{j,n}} & \frac{\partial F^{-1}_j(\Phi(x_{j,n}))}{\partial x_{j,n}} \\
\frac{\partial F^{-1}_i(\Phi(x_{i,n+\tau}))}{\partial y_{j,n}} & \frac{\partial F^{-1}_j(\Phi(x_{j,n}))}{\partial y_{j,n}} & \frac{\partial F^{-1}_j(\Phi(x_{j,n}))}{\partial x_{j,n}} \\
\end{vmatrix}
\end{align*}
\]

(3.10)

By the calculus identity, we obtain

\[
\frac{\partial F^{-1}_i(\Phi(x_{i,n+\tau}))}{\partial x_{i,n+\tau}} = \frac{\partial F^{-1}_i(\Phi(x_{i,n+\tau}))}{\partial \Phi(x_{i,n+\tau})} \frac{\partial \Phi(x_{i,n+\tau})}{\partial x_{i,n+\tau}} = \frac{1}{f_i(F^{-1}_i(\Phi(x_{i,n+\tau})))} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x_{i,n+\tau}^2}{2}\right)
\]

(3.11)

By the same procedure as (3.11), we obtain

\[
\frac{\partial F^{-1}_j(\Phi(x_{j,n}))}{\partial x_{j,n}} = \frac{1}{f_j(F^{-1}_j(\Phi(x_{j,n})))} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x_{j,n}^2}{2}\right)
\]

(3.12)

Substituting (3.11) and (3.12) into (3.10), we obtain

\[
\begin{align*}
|J(Y, X)|_{x_{i,n+\tau} = y_{i,n+\tau}}^{x_{i,n} = y_{i,n}} &= \frac{1}{f_i(F^{-1}_i(\Phi(x_{i,n+\tau})))} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x_{i,n+\tau}^2}{2}\right) \frac{1}{f_j(F^{-1}_j(\Phi(x_{j,n})) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x_{j,n}^2}{2}\right)} \\
&= \frac{1}{f_i(y_{i,n+\tau})} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(\Phi^{-1}(F_i(y_{i,n+\tau})))^2}{2}\right) \frac{1}{f_j(y_{j,n})} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(\Phi^{-1}(F_j(y_{j,n})))^2}{2}\right)
\end{align*}
\]

(3.13)

Substituting (3.8) and (3.13) into (3.9), we obtain
Simplifying (3.14), we obtain

$$f_{i,j}(y_{i,n+\tau}, y_{j,n}) = f_{i}(y_{i,n+\tau})f_{j}(y_{j,n}) \frac{1}{\sqrt{2\pi \text{cov}_{y,y}}} \exp \left\{ -\frac{\text{cov}_{y,y}^{-1} \left[ \Phi^{-1}(F_{i}(y_{i,n+\tau})) \right]^{\top} \left[ \begin{array}{c} 1 \\ r_{y,y} \end{array} \right]^{-1} \left[ \Phi^{-1}(F_{j}(y_{j,n})) \right]}{2} \right\} \times \frac{1}{\sqrt{2\pi \text{cov}_{y,y}}} \exp \left\{ -\frac{\text{cov}_{y,y}^{-1} \left[ \Phi^{-1}(F_{i}(y_{i,n+\tau})) \right]^{\top} \left[ \begin{array}{c} 1 \\ r_{y,y} \end{array} \right]^{-1} \left[ \Phi^{-1}(F_{j}(y_{j,n})) \right]}{2} \right\}$$

(3.14)

Denoting the variance of the $i$-th fading envelope process, $y_{i,n}$, as $\sigma_{i}^2$, by the definition of correlation coefficients, we have

$$\rho_{y,y} = \frac{1}{\sigma_{i}\sigma_{j}} \int_{0}^{\infty} \int_{0}^{\infty} (y_{i,n+\tau} - \mu_{i})(y_{j,n} - \mu_{j}) f_{i,j}(y_{i,n+\tau}, y_{j,n}) dy_{i,n+\tau} dy_{j,n}.$$  

(3.16)

Substituting (3.14) into (3.16), we obtain
\[
\rho_{ij,\tau} = \frac{1}{\sigma_i \sigma_j} \int_0^\infty \left( (y_{i,n+\tau} - \mu_i)(y_{j,n} - \mu_j) \right) \frac{f_i(y_{i,n+\tau}) f_j(y_{j,n})}{\left[ \begin{array}{c} 1 \\ r_{ij,\tau} \end{array} \right]^2} \\
\times \exp \left\{ \frac{\left( \Phi^{-1}(F_i(y_{i,n+\tau})) \right)^2}{2} + \frac{\left( \Phi^{-1}(F_j(y_{j,n})) \right)^2}{2} - \left[ \Phi^{-1}(F_i(y_{i,n+\tau})) \right]^T \left[ \begin{array}{c} 1 \\ r_{ij,\tau} \end{array} \right]^{-1} \left[ \Phi^{-1}(F_j(y_{j,n})) \right] \right\} \\
\times dy_{i,n+\tau} dy_{j,n}.
\]

(3.17)

Given explicit values of \(\rho_{ij,\tau}, f_i(.)\), \(F_i(.)\), and other relevant parameters, various iterative numerical methods can be used to solve for \(r_{ij,\tau}\) by using (3.17). Our own experiences show accurate numerical solutions can be obtained readily as shown in the following three examples. In the above derivations, the \(i\) and \(j\) are allowed to be equal. Therefore, (3.17) includes both ACFs, i.e., \(i = j\), and CCFs, i.e., \(i \neq j\). It is noted that the \(r_{ij,\tau}\) and \(\rho_{ij,\tau}\) are equal to 1 for \(\forall (i, j, \tau) \in \{i, j, \tau \mid (i = j) \cap (\tau = 0)\}\). Thus, we only need to evaluate \(r_{ij,\tau}\) from \(\rho_{ij,\tau}\) using (3.17) for \(\forall (i, j, \tau) \not\in \{i, j, \tau \mid (i = j) \cap (\tau = 0)\}\).

3.2.3. Summary of the Proposed Approach

For the given \(\rho_{ij,\tau}, F_i(y_i),\) and \(f_i(y_i)\), perform the following:

1) Numerically evaluate the \(r_{ij,\tau}\) corresponding to the given \(\rho_{ij,\tau}, F_i(y_i),\) and \(f_i(y_i)\) using (3.17); 2) Evaluate the parameters, \(A_i\) and \(Q_i\) of the Gaussian vector AR by (3.5) and (3.6); 3) Generate samples, \(x_{i,n}\), from the Gaussian vector AR from \(A_i\) and \(Q_i\); 4)
Perform \( y_{i,n} = F_i^{-1}(\Phi(x_{i,n})) \). The generated \( y_{i,n} \) have the desired processes with the desired \( \rho_{y,z}, F_i(y_i), \) and \( f_i(y_i) \).

### 3.3. Properties and Discussions

In the design of the proposed approach, we used \( r_{i,i}=1 \) for \( \forall (i,j,\tau) \in \{i,j,\tau \mid (i=j) \land (\tau = 0)\} \) in the Yule-Walker equations. The variances, i.e., \( r_{n,0}^2 \ \forall \ i, \) of the generated individual processes in the Gaussian vector AR process are all equal to 1. In other words, the marginal pdfs of the processes in the Gaussian vector AR process generated by this approach are normalized Gaussian pdfs. Therefore, for the normalized Gaussian cdf \( \Phi(\cdot) \) and normalized Gaussian \( x_i, \) the inverse transform sampling theorem [3.22] states that the \( \Phi(x_{i,n}) \) is uniformly distributed. Since \( \Phi(x_{i,n}) \) is uniform distribution, by the inverse sampling transform theorem [3.22], the \( y_{i,n} = F_i^{-1}(\Phi(x_{i,n})) \) is the random variable distributed by the cdf \( F_i(\cdot), \) with its pdf \( f_i(\cdot). \)

The choices of \( F_i(\cdot) \) in the inverse sampling transform, i.e., \( y_{i,n} = F_i^{-1}(\Phi(x_{i,n})) \) in Fig. 3.1., allow the output pdfs to be heterogeneous. Therefore, by choosing various \( F_i(\cdot) \) for different \( i, \) our proposed approach is able to generate correlated fading envelope processes of heterogeneous pdfs.

The solution of Gaussian vector AR parameters in (3.5), \( A=R_{xx}^{-1}V, \) sometimes produces unstable Gaussian vector AR or the unrealizable \( Q \) due to the ill-conditioned \( R_{xx} \) matrix. To avoid this problem, we adopt a regularization approach [3.2][3.3], i.e.,
using \( A = \left( R_{ss} + \varepsilon I \right)^{-1} V \), where \( \varepsilon \) is a small number and \( I \) is the identity matrix. The choices of \( \varepsilon \) are considered in [3.3].

3.4. Simulations

Three examples are demonstrated in this section. The first example is a scenario with a single auto-correlated Nakagami channel [3.19]. The second example is a 2x2 MIMO scenario, where 4 Rayleigh channels are generated [3.2]. The parameters and simulation settings of the above two examples are selected from previously published results in [3.2] and [3.19]. The third example is a scenario with 3 channels of heterogeneous pdfs, i.e., the 3 channels with Nakagami, Rician, and Rayleigh pdfs individually. The parameters of the third example are selected to demonstrate the capability of generating correlated heterogeneous channel envelopes.

3.4.1. Example 1: The Single Nakagami Channel

In this example, the settings and parameters of this example are selected from [3.19]. The Nakagami pdf and cdf are respectively expressed as

\[
f_{\text{Naka}}(y) = \frac{2m^m}{\Gamma(m)\omega^m} y^{2m-1} \exp\left(-\frac{m}{\omega}y^2\right), \quad F_{\text{Naka}}(y) = \mathcal{F}(m, \frac{m}{\omega}y^2),
\]

where \( \Gamma(\cdot) \) is the Gamma function and \( \mathcal{F}(\cdot, \cdot) \) is the regularized incomplete Gamma function. The mean of the Nakagami random variable described by (3.18) is

\[
\frac{\Gamma(m+1/2)}{\Gamma(m)} \left( \frac{\omega}{m} \right)^{1/2}.
\]

The parameters are chosen as \( m=1.69 \) and \( \omega=1.16 \) so that the Nakagami process has the mean of 1. The theoretical, i.e., the desired, ACF is chosen as
\[ \rho_{\text{Naka}, \tau} = J_0^2(2\pi f_D \Delta |\tau|) \], where the \( \tau \) represents the index of the time lag, the \( \Delta \) represents the sampling interval, the Doppler shift is 
\[ f_D = \frac{v}{\lambda} = \frac{vf_c}{3 \times 10^8} \text{ Hz}, \quad v = 27.78 \text{ m/s}. \]

There are two settings for the sampling rates, i.e., the low sampling rate and the high sampling rate. In the low sampling rate scenario, the carrier frequency is \( f_c = 1.8 \text{ GHz} \) and the sampling rate is 9,600 Hz, which corresponds to \( \Delta = 1/9600 \) second. In the high sampling rate scenario, the carrier frequency is \( f_c = 900 \text{ MHz} \) and the sampling rate is 24,300 Hz, which corresponds to \( \Delta = 1/24300 \) second.

The empirical and theoretical ACFs are shown in Fig. 3.2. Comparing the empirical ACFs with the theoretical ACFs, it is observed that the empirical ACFs closely follow the theoretical ACFs. The empirical cdfs of the generated fading envelope samples and the corresponding theoretical cdfs are compared in Fig. 3.3, where the empirical cdfs closely follow the theoretical cdfs except at the tails. To further verify the empirical cdfs, we conducted Kolmogorov-Smirnov [3.30] tests (KSTs) on the generated fading envelopes with their respective theoretical cdfs. The resulting p-values of the KSTs are 0.104 and 0.375, which pass the KSTs with significance level of 0.05, respectively for the low sampling rate and high sampling rate scenarios. The observations of the cdfs in Fig. 3.3 and the KST results indicate the achievement of the design goal that the targeted distribution is the Nakagami distribution. The observations in Fig. 3.2 and 3.3 and the KST results verify the capabilities of the proposed approach to generate the desired auto-correlated Nakagami fading envelopes.
Figure 3.2. The ACFs of Nakagami channels in the low sampling rate (9600 Hz) and the high sampling rate (24300 Hz).

Figure 3.3. The empirical and theoretical cdfs of Nakagami channels in the low sampling rate (9600 Hz) and the high sampling rate (24300 Hz).
3.4.2. Example 2: The 2x2 Rayleigh MIMO channels

In this 2x2 Rayleigh MIMO example, there are a total of four Rayleigh channels, with parameters selected from [3.2]. The Rayleigh pdf and cdf are expressed as

\[
 f_{\text{Ray}}(y) = \frac{y \exp\left(\frac{-y^2}{2\sigma_{\text{Ray}}^2}\right)}{\sigma_{\text{Ray}}^2}, \quad F_{\text{Ray}}(y) = 1 - \exp\left(\frac{-y^2}{2\sigma_{\text{Ray}}^2}\right). \tag{3.19}
\]

The parameter is chosen as \( \sigma_{\text{Ray}} = 0.7979 \) for all the four envelope processes such that the four envelope processes have their means equal to 1. The ACFs and CCFs of the four channels are chosen as

\[
 \rho_{11,\tau} = \rho_{22,\tau} = \rho_{33,\tau} = \rho_{44,\tau} = J_0(2\pi f_D \Delta |\tau|), \tag{3.20a}
\]

\[
 \rho_{12,\tau} = \rho_{34,\tau} = J_0\left(\left\{a^2 + b^2 - 2ab \cos(\beta - \gamma)\right\}^{\frac{1}{2}}\right), \tag{3.20b}
\]

\[
 \rho_{13,\tau} = \rho_{24,\tau} = J_0\left(\left\{a^2 + c^2 \eta^2 - 2ab \eta \sin(\alpha) \sin(\gamma)\right\}^{\frac{1}{2}}\right), \tag{3.20c}
\]

\[
 \rho_{14,\tau} = \rho_{23,\tau} = J_0\left(\left\{a^2 + b^2 + c^2 \eta^2 - 2ab \cos(\beta - \gamma) - 2c\eta \sin(\alpha) [a \sin(\gamma) - b \sin(\beta)]\right\}^{\frac{1}{2}}\right), \tag{3.20d}
\]

where \( f_D \Delta \) is the maximum Doppler frequency shift normalized by the sampling time \( \Delta \), and the \( \tau \) represents the index of the time lag. The parameters are \( a = 2\pi f_D \Delta |\tau|, \)
\( b = 2\pi d / \lambda, \) and \( c = 2\pi \delta / \lambda. \) The geometric interpretations of these parameters in the propagation environment are detailed in [3.2] and [3.4]. The values of the parameters are chosen as \( f_D \Delta = 0.045, \ d / \lambda = 0.5, \ \delta / \lambda = 17, \ \eta = 4^\circ, \ \alpha = \beta = 90^\circ, \) and \( \gamma = 0^\circ \) [3.2]. The empirical and theoretical ACFs and CCFs are shown in Fig. 3.4 and 3.5. Comparing
the empirical ACFs and CCFs with their theoretical counterparts, it is observed that the empirical ACFs and CCFs closely follow their theoretical counterparts. The empirical cdfs and the theoretical cdfs are compared in Fig. 3.6, where the empirical cdfs closely follow the theoretical cdfs. Besides observing the empirical and theoretical cdfs, we conducted KSTs on the generated fading envelopes with their respective theoretical cdfs. The resulting p-values of the KSTs are 0.057, 0.49, 0.32, and 0.098 respectively for the four Rayleigh channels, which pass the KSTs with significance level of 0.05. The observations of the cdfs in Fig. 3.6 and the KSTs show the generated fading envelopes closely follow the targeted Rayleigh distributions. The observations in Fig. 3.4-3.6 and the KST results verify the effectiveness of the proposed approach in generating the desired multiple fading envelopes of Rayleigh pdfs.
Figure 3.4. The ACFs of the 2x2 MIMO scenario.

Figure 3.5. The CCFs of the 2x2 MIMO scenario.
3.4.3. Example 3: The Three Envelope Processes with Nakagami, Rician, and Rayleigh pdfs

In this example, there are three envelope processes each having the Nakagami, Rician, and Rayleigh pdfs respectively. This scenario is designed to demonstrate the capability of the proposed approach to generate envelope processes of different pdfs. The Rician pdf and cdf are expressed as

\[ f_{\text{Rician}}(y) = \frac{y}{\sigma_R^2} \exp\left(-\left(\frac{y^2 + \nu^2}{2\sigma_n^2}\right)\right) I_0\left(\frac{\nu y}{\sigma_R^2}\right), \quad F_{\text{Rician}}(y) = 1 - Q\left(\frac{\nu y}{\sigma_R^2}, \frac{y}{\sigma_R}\right), \quad (3.21) \]
where $I_0(\bullet)$ is the modified Bessel function of the first kind with order zero and $Q(\bullet, \bullet)$ is the Marcum Q-function. In the settings of this example, the first envelope process is the Nakagami process with its pdf (3.18) parameterized by $m = 2.5$ and $\omega = 1$, such that its mean is equal to 0.95 and its variance is equal to 0.095. The second envelope process is the Rician process with its pdf (3.21) parameterized by $\sigma_{Ri} = 1$ and $\nu_{Ri} = 1$, such that its mean is equal to 1.55 and its variance is equal to 0.602. The third envelope process is the Rayleigh process with its pdf (3.19) parameterized by $\sigma_{Ray} = 1$, such that its mean is equal to 1.25 and its variance is equal to 0.43. The settings and parameters are intentionally chosen to create 3 fading envelope processes with different means, variances, and pdfs. The ACFs and CCFs are chosen as the exponential type [3.25][3.26][3.27], expressed as

$$\rho_{11,\tau} = \rho_{22,\tau} = \rho_{33,\tau} = \exp(-f_D \Delta|\tau|),$$

$$\rho_{12,\tau} = \rho_{21,\tau} = \rho_{13,\tau} = \rho_{31,\tau} = \rho_{23,\tau} = \rho_{32,\tau} = \kappa\exp(-f_D \Delta|\tau|),$$

where $\tau$ represents the index of the time lag and the $f_D \Delta$ is the decay constant of the correlations. The parameters are chosen as $f_D \Delta = 0.4$ and $\kappa = 0.6$. The generated and theoretical ACFs and CCFs are shown in Fig. 3.7 and 3.8. Comparing the empirical ACFs and CCFs with their theoretical counterparts, it is observed that the empirical ACFs and CCFs closely follow the theoretical counterparts. The empirical cdfs and the theoretical cdfs are compared in Fig. 3.9. The empirical cdfs are observed to closely follow the theoretical cdfs. Besides observing the empirical and theoretical cdfs, we conducted KSTs on the generated fading envelopes with their respective theoretical cdfs. The
resulting p-values of the KSTs are 0.086, 0.073, and 0.23, respectively for the Nakagami, Rician, and Rayleigh channels, which pass the KSTs with significance level of 0.05. The observations of the cdfs in Fig. 3.9 and the KST results show that the generated fading envelopes of the three channels closely follow Nakagami, Rician, and Rayleigh distributions respectively. The observations in Fig. 3.7-3.9 and the KST results demonstrate the capabilities of the proposed approach to generate the desired auto-correlated and cross-correlated heterogeneous fading envelope processes. In Fig. 3.10, the empirical and theoretical un-normalized ACFs are shown to close to each other.

Figure 3.7. The ACFs of the Nakagami, Rician, and Rayleigh channels.
Figure 3.8. The CCFs of the Nakagami, Rician, and Rayleigh channels.

Figure 3.9. The empirical cdfs and theoretical cdfs of the Nakagami, Rician, and Rayleigh channels.
3.5. References


Chapter 4. Decision Fusion in Sensor Networks for Spectrum Sensing based on Likelihood Ratio Tests

4.1. Introduction

The cognitive radio facilitates the increased utilization of the spectra. In conventional spectrum allocation schemes, the spectra are licensed to a certain group of users or systems, preventing other users from using the spectra. These conventional spectrum allocation schemes lead to low utilization of the spectra. To increase the utilization of the spectra, the concept of the cognitive radio is to allow the secondary users to use the spectra when the channel is not occupied by the primary (i.e. licensed) users.

Several techniques [4.1] have been used for spectrum sensing, such as the energy detection, the matched filter detection, and the feature detection. These techniques are employed on the single-user detection scenario.

One approach to increase the sensing accuracies is to adopt cooperative spectrum sensing [4.2][4.3], where the distributed secondary users form the sensor network and make independent detections (i.e. binary hypothesis tests), and then transmit their decisions to the fusion center (FC). The fusion center makes the final decision based on the decisions from the secondary users. The goal of the cooperative spectrum sensing is to make the decisions, whether to use the intended channel or not, for the secondary users based on the final decision of the fusion center. Thus, the fusion rule at the fusion center must be well designed, such that the probability of false alarm and the probability of detection of its final decision achieve the performance criteria. The cooperative sensing techniques in the cognitive radio belong to the category of the distributed detections.
[4.4]-[4.7]. We suggest the lowered-bounded probability of detection (LBPD) criterion to be more suitable than the Neyman-Pearson (NP) criterion for the cognitive radio. We provide algorithms to explicitly compute the optimal decision fusion rules for the two criteria.

Since the fading channels are ubiquitous in wireless communications, we investigate the single-sensor detection and collaborative-detection under the influences of the fading channels. We formulate the pdfs of the fading channel gains into the fusion rules. Our approaches, called the likelihood ratio test with fading statistics (LRFS), incorporate the fading statistics into the likelihood ratio test framework. The receiver operating curves (ROCs) are evaluated. The ROCs show performance improvements when the fading statistics are incorporated in the detection frameworks. For the clear presentations, the notations defined in this chapter are self-contained and infer to this chapter only. Notations outside of this chapter are defined in the respective chapters.

4.2. Collaborative Decision Fusion for Spectrum Sensing

The spectrum sensing scenario by the sensor networks is described in the following. The cognitive radio network with $N$ secondary users is deployed. The $N$ secondary users form the sensor network to detect the spectrum holes. Each secondary user conducts local detections using the energy detection, the matched filter detection, or the feature detection. Each secondary user decides either $H_0$ (channel vacant) or $H_1$ (channel occupied). The probabilities of the local false alarm are denoted as $\{P_{fa_i} \mid i = 1, 2, \ldots, N\}$, where $P_{fa_i}$ represents the probability of false alarm of the $i$-th user. The probabilities of
detection are denoted as \( \{ P_{Di} \}_{i=1,2,\ldots,N} \), where \( P_{Di} \) represents the probability of detection of the \( i \)-th user. The local decisions of the users are denoted as \( \{ U_i \}_{i=1,2,\ldots,N} \).

The \( U_i \) takes the value -1 when the local decision of the \( i \)-th user is the hypothesis \( H_0 \), meaning the channel is vacant. The \( U_i \) takes the value 1 when the local decision of the \( i \)-th user is the hypothesis \( H_1 \), meaning the channel is occupied by the primary users. The \( \{ P_{Di} \}_{i=1,2,\ldots,N} \), \( \{ U_i \}_{i=1,2,\ldots,N} \) are abbreviated as \( \{ P_{Di} \} \), \( \{ U_i \} \) respectively. All possible realizations of \( \{ U_i \} \) constitute the space containing all possible \( N \)-bit sequences, denoted as \( \{-1,1\}^N \). The realization of \( \{ U_i \}_{i=1,2,\ldots,N} \) is denoted as \( \{ u_i \}_{i=1,2,\ldots,N} \), abbreviated as \( \{ u_i \} \).

The \( \{ P_{Di} \} \) and \( \{ P_{Ui} \} \) are fixed and known to the fusion center. When the cooperative spectrum sensing mechanism is activated, the FC obtains the realized local decisions, \( \{ u_i \} \). The FC makes the final decision, \( U_f \), which takes value -1 (i.e. \( H_0 \)) or 1 (i.e. \( H_1 \)), by the fusion rule \( f_0(\{ u_i \}) \), defined as \( f_0 : \{-1,1\}^N \rightarrow \{-1,1\} \). Since the randomized fusion rules include the deterministic fusion rule as the special case, we consider \( f_0 \) in the category of the randomized fusion rules. The proposed algorithm is to design the \( f_0 \) based on the given \( \{ P_{Di} \} \) and \( \{ P_{Ui} \} \), such that the false alarm probability of the \( U_f \), denoted as \( P_{FAf} \), and probability of detection of the \( U_f \), denoted as \( P_{Df} \), fulfill the specified criteria. The criteria include Neyman-Pearson Criterion and Lower-Bounded Probability of Detection.

4.2.1. Neyman-Pearson Criterion
The Neyman-Pearson criterion (NP criterion) is aimed to maximize the probability of detection under the condition that the probability of false alarm is below the predefined threshold, denoted as $\alpha$. Applying the NP criterion to the cooperative spectrum sensing, the goal is to design a $f_0$ such that $P_{D_{fc}}$ is maximized, under the condition that $P_{FA_{fc}} < \alpha$.

4.2.2. Lower-Bounded Probability of Detection

The possible interferences and channel conflicts to the primary users cause severe concerns. Specifically, when the channel is occupied by the primary users (i.e. $\mathcal{H}_1$) but the secondary users decide $\mathcal{H}_0$, the channel conflicts happen. When the channel is vacant (i.e. $\mathcal{H}_0$) but the secondary users decide $\mathcal{H}_1$, the secondary users miss the opportunity of using the spectrum hole. Therefore, from the viewpoint of limiting the probability of channel conflicts to protect the primary users, the desired criterion is to maintain the probability of detection above a pre-defined value, while minimizing the probability of false alarm (i.e. missed opportunities). Following this viewpoint, we suggest the lower-bounded probability of detection criterion (LBPD criterion). In LBPD criterion, the goal is to minimize the probability of false alarm under the condition that the probability of detection is lower-bounded by the predefined threshold, $\beta$. This criterion guarantees the channel occupation by primary users being detected above the predefined probability. Therefore, the probability of channel conflicts is kept below a certain probability. Applying the LBPD criterion in the cooperative spectrum sensing, the goal is to design a $f_0$ that $P_{FA_{fc}}$ is minimized, under the condition that $P_{D_{fc}} > \beta$.

4.3. Computing the Decision Fusion Rule
In this section, we create an algorithm to compute the optimal fusion rule under the given \( \{P_{1,u}\} \) and \( \{P_{2,u}\} \) of the secondary users without fading influences. The \( f_0 \) is generated by separating the \( \{-1,1\}^N \) into three mutually exclusive subsets, denoted as \( S_0 \), \( S_1 \), and \( r \), which satisfy \( S_0 \cup S_1 \cup r = \{-1,1\}^N \). The probability, \( \epsilon \), associated with \( r \), is used by \( f_0 \) to randomly generate the decision when the input is \( r \). The \( f_0 \) is summarized as

\[
f_0({u_i}) = \begin{cases} 
  -1, & \text{when } {u_i} \in S_0 \\
  +1, & \text{when } {u_i} \in S_1 \\
  +1 \text{ with probability } \epsilon, & \text{when } {u_i} \equiv r \\
  -1 \text{ with probability } 1 - \epsilon, & \text{when } {u_i} \equiv r.
\end{cases}
\] (4.1)

Each element in \( \{-1,1\}^N \) represents a possible realization of the \( \{U_i\} \). Thus, each element has its associated probabilities of occurrences under \( H_0 \) and \( H_1 \). For each element \( \gamma_j \in \{-1,1\}^N \), we denote \( P(\gamma_j | H_0) \) by \( P_{\gamma_j, H_0} \) and \( P(\gamma_j | H_1) \) by \( P_{\gamma_j, H_1} \). The \( P_{\gamma_j, H_0} \) and \( P_{\gamma_j, H_1} \) can be computed by the given \( \{P_{1,u}\} \) and \( \{P_{2,u}\} \). The likelihood ratio, associated with \( \gamma_j \), is defined as \( \Lambda_j = \frac{P_{\gamma_j, H_1}}{P_{\gamma_j, H_0}} \). The algorithms for computing the fusion rules under the NP criterion and the LBPD criterion are stated separately in Table 1.
## Algorithm for NP Criterion

**Input:** \( \alpha, \{ \gamma_j \}, \{ P_{\gamma_j|h_0} \}, \{ P_{\gamma_j|h_1} \}, \{ \Lambda_j \} \).

**Output:** \( S_0, S_1, \varepsilon, r. \)

**Initialization:** \( T_{FA} = 0, \quad S_0 = \emptyset, \quad S_1 = \emptyset. \)

1. \( k := \arg \max_j \{ \Lambda_j \}; \)
2. \( \text{if } (T_{FA} + P_{\gamma_j|h_0}) < \alpha \text{ then} \)
   \( S_1 := S_1 \cup \gamma_k; \)
   \( \Lambda_k := 0; \)
   \( T_{FA} := T_{FA} + P_{\gamma_j|h_0}; \)
   \( \text{Go to step 1;} \)
3. \( \varepsilon = \frac{\alpha - T_{FA}}{P_{\gamma_j|h_0}}; \quad r = \gamma_k; \)
   \( S_0 := \{0,1\}^N \setminus \{ S_1 \cup r \}; \)
4. **Return** \( S_0, S_1, \varepsilon, r. \)

## Algorithm for LBPD Criterion

**Input:** \( \beta, \{ \gamma_j \}, \{ P_{\gamma_j|h_0} \}, \{ P_{\gamma_j|h_1} \}, \{ \Lambda_j \} \).

**Output:** \( S_0, S_1, \varepsilon, r. \)

**Initialization:** \( T_D = 0, \quad S_0 = \emptyset, \quad S_1 = \emptyset. \)

1. \( k := \arg \max_j \{ \Lambda_j \}; \)
2. \( \text{if } (T_D + P_{\gamma_j|h_1}) < \beta \text{ then} \)
   \( S_1 := S_1 \cup \gamma_k; \)
   \( \Lambda_k := 0; \)
   \( T_D := T_D + P_{\gamma_j|h_1}; \)
   \( \text{Go to step 1;} \)
3. \( \varepsilon = \frac{\beta - T_D}{P_{\gamma_j|h_1}}; \quad r = \gamma_k; \)
   \( S_0 := \{0,1\}^N \setminus \{ S_1 \cup r \}; \)
4. **Return** \( S_0, S_1, \varepsilon, r. \)

### Table 4.1 Pseudo-codes for computing the decision fusion rules.

The notations are defined as \( \emptyset \) meaning empty set, \( \cup \) meaning set union, := meaning assigning the right-hand-side to the left-hand-side, and \( \setminus \) meaning the set difference.

The rationale of the algorithm for the NP criterion is to view the given \( \alpha \) as the scarce resource, which must be consumed sparingly by choosing the \( \gamma_j \) with the largest \( \Lambda_j \) into
$S_1$. Opposed to the NP criterion, the rationale of the algorithm for the LBPD criterion is to view the given $\beta$ as the threshold which must be achieved quickly by choosing the $\gamma_j$ with the largest $\Lambda_j$ into $S_1$. The randomization by the $\epsilon$ associated with $r$ is to utilize the gap between the predefined threshold of the criteria and the achievable $P_{FAc}$ (NP criterion) or $P_{Dc}$ (LBPD criterion) by the deterministic fusion rules.

For the generated fusion rule, the probability of false alarm and the probability of detection at the fusion center are $P_{FAc} = \alpha$ and $P_{Dc} = \epsilon \cdot p_{H_1} + \sum_{\{j | \gamma_j \in S_1\}} p_{H_1}$ for the NP criterion, and $P_{FAc} = \epsilon \cdot p_{H_0} + \sum_{\{j | \gamma_j \in S_1\}} p_{H_0}$ and $P_{Dc} = \beta$ for the LBPD criterion. In other words, for the specific $\alpha$, $\beta$, $\{P_{FAi}\}$, and $\{P_{Di}\}$, the algorithm generates a fusion rule and its associated pair of false alarm probability and detection probability, denoted as $(P_{FAc}, P_{Dc})$. The pair, $(P_{FAc}, P_{Dc})$, is the receiver operating point (ROP) corresponding to that specific fusion rule.

To find all the feasible ROPs of the fusion center under the proposed algorithm, the algorithm can be routinely performed on values of $\alpha$, incremented from 0 to 1. In other words, by gradually increasing the $\alpha$ from 0 to 1 and performing the algorithm repeatedly on each specific value of $\alpha$, the feasible fusion rules and ROPs can be found. The set of the feasible ROPs forms the receiver operating curve (ROC). In the LBPD criterion, the ROC can be found by performing the same procedures with $\beta$ incremented from 0 to 1. Our numerical results show that the ROCs generated by NP criterion are the same as the ROCs generated by the LBPD criterion, as demonstrated in Fig. 4.1-4.4.
Although the ROCs of the NP algorithm is the same as the ROCs of the LBPD algorithm, for arbitrary $\alpha$ and $\beta$, the two algorithms generate different fusion rules. The LBPD criterion fits the concept of the cognitive radio, whose first priority is to protect the primary users above a certain quality of service while allowing the secondary users to utilize the vacant channels.

4.4. Detection and Decision Fusion with Fading Statistics

In this section, we first consider the detection at the single sensor under the fading channels. Besides the detection at the single sensor, the collaborative decision fusions under the fading channels by multiple sensors are considered. The explicit expressions of the test statistics are derived.

Single sensor detection under fading channels

We consider the scenario where the pdf of the channel gain is known at each of the secondary users. In this section, our goal is to incorporate the pdf of the fading channel gain in the likelihood ratio test of the local detector, resulting in the LRFS. We assume a slow-flat fading channel with the channel gain, denoted as $\kappa$, resulting in a multiplicative gain on the transmitted signal. The received signal at a single detector is given by

$$X = \begin{cases} \frac{N_i H_0}{\kappa S + N_i H_i}, \\ \frac{N_i H_0}{\kappa S + N_i H_i} \end{cases} \quad (4.2)$$

where $X$ is a $m \times 1$ vector of the received signal, $N$ is $m \times 1$ white-Gaussian-noise vector with $m \times 1$ zero mean vector $\mathbf{0}$ and $m \times m$ covariance matrix $\sigma_n^2 \mathbf{I}$ ($\sigma_n^2$ is the scalar noise variance and $\mathbf{I}$ is the $m \times m$ identity matrix), $S$ is the $m \times 1$ transmitted signal known at the detector, and $\kappa$ is the scalar of the fading amplitude with its pdf,
\( p(\kappa) \), known at the detector. In this chapter, \( N(X; \mu, \Sigma) \) denotes Gaussian pdf with \( m \times 1 \) vector argument \( X \), \( m \times 1 \) mean vector \( \mu \), and the \( m \times m \) covariance matrix \( \Sigma \). The likelihood function of \( X \) conditioned on \( H_0 \) is

\[
    p(X | H_0) = N(X; \mu, \Sigma) = \frac{1}{(2\pi)^{m/2} \sigma_n^m} e^{-\frac{1}{2} (X - \mu)^T S (X - \mu)/\sigma_n^2}, \tag{4.3}
\]

where \( T \) is the transpose operator. The likelihood function of \( X \) conditioned on \( H_1 \) is

\[
    p(X | H_1) = \int_0^\infty p(X | \kappa, H_1) p(\kappa | H_1) d\kappa
    = \int_0^\infty N(X; \kappa \Sigma, \sigma_n^2 I) p(\kappa) d\kappa
    = \int_0^\infty \frac{1}{(2\pi \sigma_n^2)^{m/2}} e^{-\frac{1}{2} (X - \kappa \Sigma)^T (X - \kappa \Sigma)/2\sigma_n^2} p(\kappa) d\kappa. \tag{4.4}
\]

In (4.4), we assume that \( \kappa \) is independent of \( H_1 \), i.e., \( p(\kappa | H_1) = p(\kappa) \).

By (4.3) and (4.4), the LRFS test statistic is expressed as

\[
    \Lambda_{LRFS}(X) = \frac{p(X | H_1)}{p(X | H_0)} = \frac{1}{(2\pi \sigma_n^2)^{m/2}} e^{-\frac{1}{2} (X - \mu)^T S (X - \mu)/\sigma_n^2} \int_0^\infty \frac{1}{(2\pi \sigma_n^2)^{m/2}} e^{-\frac{1}{2} (X - \kappa \Sigma)^T (X - \kappa \Sigma)/2\sigma_n^2} p(\kappa) d\kappa
    = \int_0^\infty e^{(2\kappa X^T - \kappa \Sigma^T \Sigma \kappa)/2\sigma_n^2} p(\kappa) d\kappa. \tag{4.5}
\]

Denoting the test decision of the single detector as \( u \), \( H_0 \) as -1, and \( H_1 \) as +1, the detection criterion with the threshold, \( \lambda_{LRFS} \), is expressed as

\[
    u = +1 \quad \Lambda_{LRFS}(X) > \lambda_{LRFS}, \quad u = -1 \quad \Lambda_{LRFS}(X) < \lambda_{LRFS}. \tag{4.6}
\]

For a Rayleigh channel, the pdf of the channel gain is
\[ p_{\text{Ray}}(\kappa; \sigma_R) = \kappa \exp\left(\frac{-\kappa^2}{2\sigma_R^2}\right) / \sigma_R^2. \]  

(4.7)

Substituting (4.7) in (4.5) and using [4.14], we obtain the explicit expression of the LRFS test statistic under the Rayleigh channel as

\[
\Lambda_{\text{LRFS}}(X) = \left[ \frac{\sigma_R^2 (X^T S)^2}{e^{2(\sigma_R^2 + \sigma^2_S) S^T S} + 2\sigma^2_S} \right] \sqrt{2\pi X^T S + 2\sigma^2_S} \frac{1}{\sigma_R^2} + \frac{S^T S}{\sigma_n^2} + \sqrt{2\pi} \frac{S^T S + \sigma^2_n}{\sigma_R^2} \right],
\]

(4.8)

where \( \text{Erf} \) is the error function.

For a Rician channel, the pdf of the channel gain is

\[
p_{\text{Rician}}(\kappa; \sigma_{Ri}, \nu) = \frac{\kappa}{\sigma_{Ri}^2} \exp\left(-\frac{(\kappa^2 + \nu^2)}{2\sigma_{Ri}^2}\right) I_0\left[\frac{\kappa \nu}{\sigma_{Ri}^2}\right].
\]

(4.9)

Substituting (4.9) in (4.5), we obtain the expression of the LRFS test statistic under the Rician channel as

\[
\Lambda_{\text{LRFS}}(X) = \frac{1}{\sigma_{Ri}^2} \int_0^\infty \kappa e^{-\frac{-\kappa^2}{2\sigma_{Ri}^2} + \frac{2\kappa X^T S - \kappa^2 S^T S}{2\sigma_{Ri}^2}} I_0\left[\frac{\kappa \nu}{\sigma_{Ri}^2}\right] d\kappa,
\]

(4.10)

where \( I_0(.) \) is the Bessel function of the first kind of order zero.

We numerically evaluate the ROCs of the Rayleigh and Rician LRFS statistics in (4.8) and (4.10). Since the pdfs of \( X \) under \( H_0 \) and \( H_1 \) can be derived to be Gaussian distributions from (4.2), together with (4.8) and (4.10), the pdfs of \( \Lambda_{\text{LRFS}}(X) \) conditioned
on $H_0$ or $H_1$, and the resulting ROCs can be obtained. Besides performance analyses, the explicit expression in (4.8) also facilitates the implementation of the detection.

We denote $E[.]$ as the expectation of the random variable. In these examples, by the signal model of (4.2), the signal-to-noise-ratio (SNR) is expressed by $\text{SNR} = E[\kappa^2]|S|^2/(n\sigma_n^2)$, where $\|S\|$ is the $l_2$-norm of $S$. Figure 5 shows the ROCs of $\text{SNR} = 0$, 5, and 10 dB, for Rayleigh and Rician channels. For evaluating the SNRs in these examples, we obtain $E[\kappa^2] = 2\sigma_r^2$ for a Rayleigh channel specified in (4.7), and $E[\kappa^2] = 2\sigma_r^2 + \nu^2$ for a Rician channel specified in (4.9).

For the performance comparison, we evaluate the conventional likelihood ratio test, which is equivalent to the matched filter (MF) approach without incorporating the fading statistics. The testing statistic and the decision criterion of the MF approach is expressed by

$$\Lambda_{MF}(X) = \begin{cases} u = +1 & \frac{X^T S}{\Lambda_{MF}} > \lambda_{MF} \\ u = -1 & \lambda_{MF} < \frac{X^T S}{\Lambda_{MF}} \end{cases} \quad (4.11)$$

The likelihood function of the MF output conditioned on $H_0$ is

$$p(\Lambda_{MF}(X) | H_0) = N(\Lambda; 0, \sigma_n^2 \|S\|), \quad (4.12)$$

where $\Lambda$ represents the argument for the pdf of $\Lambda_{MF}(X)$.

The MF output conditioned on $H_1$ is

$$(\Lambda_{MF}(X) | H_1) = (\kappa S + N)^T S = \kappa S^T S + N^T S. \quad (4.13)$$
In (4.13), under a Rayleigh channel, because $\kappa \Sigma^T S$ is independent from $N^T S$, we obtain

$$p(\Lambda_MF(X) | H_1) = p_{Ray}(\Lambda; (\Sigma^T S) \sigma_R^2) * N(\Lambda; 0, \sigma^2_n \|S\|),$$  \hspace{1cm} (4.14)

where $*$ denotes the convolution operator and $\Lambda$ represents the argument for the pdf. Therefore, the ROCs of the MF approach can be analytically evaluated from (4.12) and (4.14).

4.5. Multi-Sensor Decision Fusion under Fading Channels

In this section, we consider the decision fusion scenario where the fusion center makes the final decision by fusing the decisions of the $N$ detectors under fading channels. Following the same notations in the above sections, the decisions of the $N$ detectors are denoted as $\{u_i | i = 1, 2, ..., N\}$, abbreviated as $\{u_i\}$. The $u_i$ represents the decision of the $i$-th detector, with $u_i = +1$ for $H_i$, and $u_i = -1$ for $H_o$. The detectors operate at the probabilities of detection, $\{P_{d_i} | i = 1, 2, ..., N\}$ (abbreviated as $\{P_{d_i}\}$), and the probabilities of false alarm, $\{P_{fa_i} | i = 1, 2, ..., N\}$ (abbreviated as $\{P_{fa_i}\}$), known to the FC. The local decisions, $\{u_i\}$, are transmitted to the FC through the noisy and fading channels, with channel gains denoted as $\{\kappa_i | i = 1, ..., N\}$ (abbreviated as $\{\kappa_i\}$). We consider the fading statistics, i.e., the joint pdf of $\{\kappa_i\}$, are known at the FC. The FC receives the faded and noise-corrupted decisions, $\{Y_i, i = 1, ..., N\}$, modeled [4.12][4.13] by $y_i = \kappa_i u_i + n_i$, where the $n_i$ represents the noise with Gaussian pdf $N(0, \sigma^2_n)$. Denoting $Y = (y_1, y_2, ..., y_N)$, the FC has to make the final decision, $U_y$, based on the received $Y$. The false alarm probability of the $U_y$ is denoted as $P_{fa_y}$, and

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probability of detection of the $U_i$ is denoted as $P_{d,U}$. The LRFS statistic at the FC can be expressed as

$$\Lambda(Y) = \frac{P(Y \mid H_1)}{P(Y \mid H_0)}.$$  \hspace{1cm} (4.15)

By the Bayes rule, we obtain

$$p(Y \mid H_i) = \sum_{\forall u \in \{-1,1\}^n} \int \left[ p(Y \mid \{ \kappa_i \}, \{ u_i \}, H_i) p(\{ \kappa_i \} \mid H_i) d\kappa_i d\kappa_2 \cdots d\kappa_N \right].$$  \hspace{1cm} (4.16)

From the independence of $\{ u_i \}$ and $\{ \kappa_i \}$, we have

$$p(\{ \kappa_i \}, \{ u_i \} \mid H_i) = p(\{ u_i \} \mid H_i) p(\{ \kappa_i \} \mid H_i) = p(\{ u_i \} \mid H_i) p(\{ \kappa_i \}).$$  \hspace{1cm} (4.17)

Substituting (4.17) into (4.16), we obtain

$$p(Y \mid H_i) = \sum_{\forall u \in \{-1,1\}^n} \int \left[ p(Y \mid \{ \kappa_i \}, \{ u_i \}, H_i) p(\{ u_i \} \mid H_i) p(\{ \kappa_i \}) d\kappa_i d\kappa_2 \cdots d\kappa_N \right].$$  \hspace{1cm} (4.18)

Following the same procedures, we obtain

$$p(Y \mid H_0) = \sum_{\forall u \in \{-1,1\}^n} \int \left[ p(Y \mid \{ \kappa_i \}, \{ u_i \}, H_0) p(\{ u_i \} \mid H_0) p(\{ \kappa_i \}) d\kappa_i d\kappa_2 \cdots d\kappa_N \right].$$  \hspace{1cm} (4.19)

Substituting (4.18) and (4.19) into (4.15), we obtain the explicit expression of the statistic and the decision rule as

$$\Lambda(Y) = \frac{\sum_{\forall u \in \{-1,1\}^n} \int p(Y \mid \{ \kappa_i \}, \{ u_i \}, H_1) p(\{ u_i \} \mid H_1) p(\{ \kappa_i \}) d\kappa_i d\kappa_2 \cdots d\kappa_N \ u_p = +1}{\sum_{\forall u \in \{-1,1\}^n} \int p(Y \mid \{ \kappa_i \}, \{ u_i \}, H_0) p(\{ u_i \} \mid H_0) p(\{ \kappa_i \}) d\kappa_i d\kappa_2 \cdots d\kappa_N \ u_p = -1} \lambda_p.$$  \hspace{1cm} (4.20)

In (4.20), the $p(Y \mid \{ \kappa_i \}, \{ u_i \}, H_1)$ and $p(Y \mid \{ \kappa_i \}, \{ u_i \}, H_0)$ can be obtained by Gaussian distribution $N(Y; K, \text{diag}(\sigma^2_u))$, where
\[ K = \begin{bmatrix} \kappa^i u_i \\ \kappa^j u_j \\ \vdots \\ \kappa^n u_n \end{bmatrix} \text{ and } \text{diag}(\sigma^j) = \begin{bmatrix} \sigma^j_{i,j} & 0 & \cdots & 0 \\ 0 & \sigma^j_{i,j} & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma^j_{i,j} \end{bmatrix}. \quad (4.21) \]

The \( p(\{u_i\} \mid H_i) \) and \( p(\{u_i\} \mid H_o) \) of (4.20) can be obtained by \( \{P_{n_0}\} \) and \( \{P_{n_o}\} \). The \( p(\{\kappa_i\}) \) in (4.20) is the joint distribution of channel gains.

4.6. Numerical Examples

4.6.1. Numerical Examples of NP-Criterion and LBPD-Criterion

For the purpose of comparisons, the \( k \)-out-of-\( N \) rule, the decision space search rule, and the exhaustive search approach are introduced. In the \( k \)-out-of-\( N \) rule, the fusion decision is \( H_i \) if the number of 1 in the \( \{u_i\} \) is larger than \( k \). The fusion decision is \( H_o \) if the number of 1 in the \( \{u_i\} \) is equal to or smaller than \( k \). In the decision space search rule, all the elements in \( \{0,1\}^N \) are assigned into \( S_0 \) or \( S_1 \) randomly. Each element has equal probability of being in \( S_0 \) or \( S_1 \). For the system with \( N \) users, there are \( 2^N \) elements in the domain of the fusion rule, \( \{0,1\}^N \). The number of possible fusion rules is the number of elements in the power set of \( \{0,1\}^N \). Therefore, it takes \( 2^{2^N} \) trials to exhaustively search all possible fusion rules for the \( N \)-user system. Because of the high complexity of \( 2^{2^N} \), we only conduct exhaustive search for the examples of \( N=3 \) and \( 4 \) (Fig. 4.1 and 4.2). For other cases, we randomly select 50000 fusion rules and plot their corresponding \( (P_{PA}, P_{DA}) \) pairs (Fig. 4.3 and 4.4). In Fig. 4.3 and 4.4, the 50000 \( (P_{PA}, P_{DA}) \) pairs from the random search are more concentrated with the increasing \( N \). Compared with the ROPs of
the exhaustive search, the ROCs of the proposed algorithm are optimal, as shown in Fig. 4.1 and 4.2.

Figure 4.1. The receiver operating points for $N=3$. The operating points, $\{(P_{Fa}, P_{De})\}$, of the individual users are (0.37,0.68), (0.27,0.70), and (0.27,0.62).

In Fig. 4.5, the ROCs of the single detector under fading channels are evaluated and compared. From the ROCs at the same SNR value, we note the LRFS results outperform the conventional MF results. For example at $SNR = 0$ dB and $P_{fa} = 0.05$, Fig. 4.5 shows the $P_{fa} = 0.24$ for MF, $P_{fa} = 0.45$ for the LRFS in the Rayleigh channel, and $P_{fa} = 0.47$ for the LRFS in the Rician channel. These examples show the performance improvements, contributed by incorporating the fading statistics into the detection scheme.

We evaluate the ROCs of the fusion rule (4.20) through Monte Carlo simulations. In the simulations, the settings are $P_{b,i} = 0.7$ and $P_{fa,i} = 0.2$ for all $i$. In this example, the $\{\kappa_i\}$ are independent Rayleigh random variables with parameters $\{\sigma_{\kappa_i}\}$. We evaluate the
ROCs for 2-sensor and 3-sensor scenarios with the Rayleigh parameters of $\sigma_R = 0.2$, 0.5, and 0.8 for all $i$. The results are shown in Fig. 4.6. At $\sigma_R = 0.5$ and $P_{FA} = 0.05$, Fig. 4.6 shows the $P_D = 0.38$ for the 2-sensor LRFS decision fusion, $P_{DFC} = 0.46$ for the 3-sensor LRFS decision fusion. The performance improvements by using more sensors can be evaluated by (4.20), as demonstrated in this example.

![Figure 4.3. ROCs for LRFS and MF detectors for Rayleigh and Rician channels.](image-url)
Figure 4.4. ROCs for decision fusion with 2 and 3 detectors under Rayleigh channels.

4.7. References


Chapter 5. Conclusions

In Chapter 2, we propose the MHSMM for characterizing the flat fading envelope process. We also provide associated parameter estimation algorithms in this model. The rationale of the MHSMM is to match various physical fading conditions into the processes of the states of the MHSMM. Thus, the MHSMM is flexible and capable of modeling the piece-wise stationary properties of the envelope process. The important properties of envelope process, including the envelope pdfs and the acfs, can be estimated by the MHSMM. In the parameter estimation scheme, the observed envelope sequence is segmented and the segments are used to estimate the model parameters. The parameters of the state envelope pdfs, the acf, and the state duration pdfs can be estimated by using non-parametric approaches or parametric approaches, depending on the availability of the...
prior knowledge. We demonstrate an example on the GSM system parameters under different physical fading conditions including mobility speeds and shadowing. The results showed acceptable accuracies for the MHSMM and the associated parameter estimation scheme. The parameters of the AFSMCM and the HMM were also estimated based on the simulation data. Comparing the estimated results of the MHSMM, the AFSMCM, and the HMM, the MHSMM is shown to provide the most flexible and accurate results in modeling this simulation scenario. Besides simulations, experimental data from two sites were collected and separated into two non-overlap sets. One set of the data was used to estimate the model parameters, while the other set of the data was used to compare and verify their accuracies. These experimental data verified the accuracies and the flexibilities of the MHSMM and the associated parameter estimation scheme.

In Chapter 3, we investigate the generation of correlated MIMO channels. The previous research efforts have focused on generating correlated fading processes of the same family. The proposed unified approach uses Gaussian vector AR process modeling and the inverse transform sampling method. We derive the equations to compute the required correlations of the Gaussian vector AR process from the intended ACFs, CCFs, and pdfs of the fading envelope processes. By controlling the correlations of the Gaussian vector AR process from the Yule-Walker equation, the output processes are driven to meet the desired ACFs, CCFs, and pdfs. Since the proposed approach is more general, our approach includes generating the correlated processes of the same family as the special cases. In the first example, the single Nakagami process is generated. In the second example, the multiple Rayleigh processes are generated. In the third example, to
demonstrate the capability to generate correlated fading envelope processes of heterogeneous pdfs, we demonstrate the generation of three correlated envelope processes each having Nakagami, Rician, and Rayleigh pdf respectively. The sample ACFs and CCFs are observed to closely follow their theoretical counterparts. The generation of the fading processes with the desired properties, i.e., the ACFs, CCFs, and pdfs, is crucial in evaluating the performances of various diversity systems. The proposed approach is capable of generating fading envelope processes with heterogeneous pdfs and desired correlations. The numerical accuracies of the proposed approach appear to be acceptable as observed in the examples.

In Chapter 4, we investigate the cooperative spectrum sensing by sensor networks. In the cognitive radio, the cooperative spectrum sensing techniques increase the sensing accuracies than the individual sensing. We suggest the LBPD criterion for the cognitive radio scenario. The LBPD criterion pursues the low probability of false alarm while maintaining probability of detection above a predefined level. Therefore, opposed to the conventional NP criterion, the LBPD criterion is aimed at protecting the primary users above a required probability of being detected while maximizing the secondary users’ probabilities of utilizing spectrum holes. Two algorithms are proposed for explicitly computing the fusion rules under the NP criterion and the LBPD criterion. The algorithms generate the fusion rules, the associated probability of false alarm, and the probability of detection for the final decision at the fusion center. The generated fusion rules are optimal under the specified criteria. We investigated the spectrum sensing in fading channels and derive explicit expressions for LRFS. Since the fading effects are
ubiquitous in wireless communications, the fading statistics provide the valuable information to improve the accuracies of the detections. The ill fading effects can also be mitigated by using more sensors to improve the detections.

Chapter 6. Future Research

6.1. Cognitive Radios

I will explore the following aspects of the cognitive radio: (1) I plan to advance the spectrum sensing mechanisms. The conventional approaches, e.g., the energy detector and matched filter approach, require the fixed data length and require the knowledge on the signal model to perform detections. My objective is to design the detector that adaptively pursues the quickest detection. As the critical requirements of the detectors, I plan to quantify its performances, including the probabilities of detection, false alarms, and average detection times. The cooperative sequential spectrum sensing by sensor networks will also be studied. (2) I plan to explore the possibilities of using the cognitive radio as the platform for wide-area mobile communications. The current wide-area mobile communications use cellular architectures, where the basestations coordinate the wireless resources and the wired backhaul network transport the long-distance packets. The current cellular systems lack spectral flexibility and require huge investments in the wireless equipments and the backhaul networks. I envision an opportunistic usage model based on cognitive radios, where the mobile users and service providers can join and leave the network dynamically with lower costs. This new model has the potentials to improve the spectral and market efficiencies. (3) I plan to investigate MIMO techniques in the cognitive radio. The potential advantages include reductions in interference and
increases in spectral efficiency. (4) I plan to study protocol enforcement mechanisms in
the cognitive radio. Since the cognitive radio allows the distributed access to spectra (as
opposed to the centralized channel allocation by the basestations in cellular systems), the
self-interested users have the incentive of pursuing higher performances by actions
violating the designated protocols, which unfairly jeopardizes other protocol-abiding
users’ performances. I will investigate the possibilities of designing a protocol violation
detector and using the sensor networks for protocol enforcement. The approach using
game-theoretical mechanism to align the users’ self-interests with the overall system
interests is also of my research interest.

6.2. Joint Detection, Estimation, and Learning in Time-Variant Environments

I plan to investigate the joint detection, estimation, and machine learning. In
conventional paradigms for detections and estimations, e.g., the likelihood-based
approaches, the signal model is assumed to be known at the detector and estimator.
However, in many real-world scenarios, the signal models constantly change. For
example in mobile communications, the fading channels between the moving transmitter
and receiver are changing over time and frequencies. The transmission strategies, e.g.,
choices in modulations, coding, power levels, frequencies, are crucial to the system
performances. As another example in deploying the sensor networks for intrusion
detections and localizations, the noise powers drift with the temperature and certain
sensor nodes fail over time, which causes the detection and localization models to be
time-variant. The objective of this research is to create a methodology for joint detection,
estimation, and learning for applications often encountered in the time-variant environments. I plan to investigate the theoretical aspects and the algorithmic aspects. The models, theoretical bounds, algorithms, and testbeds for specific applications will be investigated. One of my methods is to initiate this research by the multi-armed bandit model, which provides the mathematical abstraction to incorporate the strategies, rewards, and model information.