A Unified Approach for Generating Cross-correlated and Auto-correlated MIMO Fading Envelope Processes

Wei-Ho Chung, Ralph E. Hudson, and Kung Yao, Life Fellow, IEEE

Abstract—Diversity techniques for various communication and MIMO systems exploit the spatial and temporal diversity attributes to mitigate the ill effects of the fading channels. To evaluate these techniques, a method to generate multiple correlated fading channels is crucial. We propose a unified approach capable of generating correlated flat-fading envelope processes with the desired auto-correlation functions, cross-correlation functions, and probability density functions (pdfs). The proposed approach utilizes the Gaussian vector autoregressive process and the inverse transform sampling techniques. Comparing to the past research focusing on generating fading channels of the same family, the novelty of the proposed approach is its capability to generate fading processes of heterogeneous pdfs. Three examples are demonstrated. In the first example, the auto-correlated Nakagami channel is generated. The second example is designed to generate correlated 2x2 MIMO Rayleigh channels. In the third example, the proposed approach generates three correlated channels having Nakagami, Rician, and Rayleigh pdfs. The settings of the first two examples are adopted from previously published results, which are intended to verify the effectiveness of our proposed approach to tackle previously known problems. The third example demonstrates the novelty of the proposed approach to generate correlated channels of heterogeneous pdfs.

Index Terms—Fading channel statistics, correlated fading envelopes, Rayleigh pdf, Rician pdf, MIMO.

I. INTRODUCTION

In wireless communication systems, the signals traverse the fading channels between the transmitter and the receiver. These fading channels degrade the performance of the wireless communications. In this paper, we focus on the frequency-flat fading channels [1]. The Jake’s model describes the auto-correlation function (ACF) of the fading process by the zeroth-order Bessel function of the first kind [2][3]. The ACFs and cross-correlation functions (CCFs) of multiple channels are studied under various propagation environments in [4][5]. Various transmission schemes under fading have been studied. For example, in [6][7], modulation schemes are studied in the Rayleigh and Nakagami fading channels. In [8], turbo codes are investigated in correlated Rayleigh fading channels. To verify these analyses by simulations, the generation of the correlated fading processes for analyses and simulations is crucial.

The probability density functions (pdf) of the fading envelopes, i.e., the amplitude of the complex envelopes of the channel, have been investigated for many propagation scenarios. The Rayleigh [9][10] distribution is the most commonly encountered pdf for the fading envelopes. If there is a dominant component, e.g., the line-of-sight component, the Rician distribution is obtained [9][10]. Besides these pdfs, the Nakagami and Weibull pdfs [10] are studied and applicable in certain propagation environments. In the multi-channel systems where each channel encounters different fading effects, the channels are possibly heterogeneous with different fading envelope pdfs.

In past research, various techniques exploiting the diversities have been investigated and proven to be useful. For example, the techniques of diversity combining [11][12] and the MIMO system [13] utilize the spatial diversity to improve performances. The performances of diversity receivers in correlated Weibull fading channels are investigated in [14]. The space-time codes [15] exploit the temporal and spatial diversities to improve performances. The performances of those techniques highly depend on the spatial and temporal correlations. In the diversity combining and MIMO systems, the temporal and spatial correlations of the multiple channels are described by the ACFs and CCFs. In this paper, the ACFs and the CCFs are collectively called correlations. Generally, the multiple processes are characterized by the correlations. In the analysis, simulation, and experimental measurements, the generation of correlated fading processes facilitates the evaluations and verifications of the performances of the multi-channel systems. Previous research has been focused on generating correlated multiple fading envelope processes with pdfs of the same family, including the correlated Rayleigh fading envelopes [16][17][18], the correlated Rician fading envelopes [2], and the correlated Nakagami processes [19][20][21]. However, comparing to the past research focusing on the pdfs of the same family, we notice the generation of correlated fading envelope processes of pdfs from heterogeneous families may be needed in some applications. For example, the multiple channels of heterogeneous families could co-exist in the scenario where the signal transmission paths encounter heterogeneous scatters, reflections, and diffractions. Therefore,
our goal is to propose a unified approach which has the capability to generate multiple envelope processes of different families. To be more specific, for any given ACFs, CCFs, and marginal pdfs, our proposed approach generates the fading envelope processes with the desired ACFs, CCFs, and the marginal pdfs. The novelty of the proposed approach is to allow the pdfs of the processes to be taken from different families.

II. THE PROPOSED APPROACH

A. Concepts and Notations

The basis of our approach utilizes the Gaussian vector autoregressive (AR) process as the driving process. Conceptually, the samples generated by the Gaussian vector AR process are processed by the inverse transform sampling [22][23] to generate the fading envelope processes with the desired pdfs. The correlations of the fading envelope processes are determined by controlling the correlations in the Gaussian vector AR process.

Properties of Gaussian AR have been extensively investigated in [22][24][3]. One of their useful properties is the matrix-valued Yule-Walker equation [2][3], which yields the parameters of the Gaussian vector AR process for the given correlations. Because of these useful and well-formulated properties, the Gaussian vector AR process is selected as the driving process in our design. The block diagram of the proposed approach is illustrated in Fig. 1. The Gaussian vector AR process, shown at the left-hand-side of Fig. 1, is used to yield the fading processes. The samples generated by the Gaussian vector AR process are transformed by the inverse transform sampling, shown in the middle blocks of Fig. 1. The output of the inverse transform sampling generates the desired fading envelopes, shown at the right-hand-side of Fig. 1. For the Gaussian vector AR process, the value of the i-th process at time n is denoted as $x_{i,n}$. For the fading envelope processes, the value of the i-th process at time n is denoted as $y_{i,n}$, where $x_{i,n}$ and $y_{i,n}$ are real numbers. The coefficients of the Gaussian vector AR process for the given ACFs, CCFs, and pdfs. The correlations of the fading envelope processes are determined by controlling the correlations in the Gaussian vector AR process.

The coefficient matrix is denoted by the $M \times p$ matrix

$$\mathbf{R}_{xx} = \begin{bmatrix} R_{xx,0} & \cdots & R_{xx,-p+1} \\ \vdots & \ddots & \vdots \\ R_{xx,p-1} & \cdots & R_{xx,0} \end{bmatrix},$$

and the $M \times M$ matrix

$$\mathbf{V} = -E \begin{bmatrix} X_{n+1}X_n^T \\ X_{n+2}X_n^T \\ \vdots \\ X_{n+p}X_n^T \end{bmatrix} = - \begin{bmatrix} R_{xx,1} \\ R_{xx,2} \\ \vdots \\ R_{xx,p} \end{bmatrix}.$$  

The matrix-valued Yule-Walker equation is expressed as

$$R_{xx} \mathbf{A} = \mathbf{V}. $$  

The $M \times M$ correlation matrix, $Q$, of the noise in the vector AR can be obtained from

$$Q = R_{xx,0} + \sum_{k=1}^{p} R_{xx,-k} \mathbf{A}_k^T. $$  

By solving (5) and (6), the matrix-valued Yule-Walker equation yields the parameters of the Gaussian vector AR process, i.e., $A_i$ and $Q$, for the specified correlation coefficients, $r_{ij,\tau}$. Since we can solve $A_i$ and $Q$ from the $r_{ij,\tau}$, the next required step is to solve for $r_{ij,\tau}$ from the desired $\rho_{ij,\tau}$ and $f_i(y_i)$. In other words, based on this framework, we need to establish the relationships among $\rho_{ij,\tau}$, $f_i(y_i)$, and $r_{ij,\tau}$, such that the $r_{ij,\tau}$ can be computed from the $\rho_{ij,\tau}$ and $f_i(y_i)$. Relationships among $r_{ij,\tau}$, $\rho_{ij,\tau}$, and $f_i(y_i)$ are given in the next section.
B. Deriving the Equations of $\rho_{ij,\tau}$, $f_i(y_i)$, and $r_{ij,\tau}$

Based on the framework in Fig. 1, we have $y_{i,n} = F_{i}^{-1}(\Phi(x_{i,n}))$, where $x_{i,n}$ represents the sample of the $i$-th process of the Gaussian vector AR process at time $n$. The joint pdf of the samples of the $i$-th process at time $n+\tau$, $x_{i,n+\tau}$, and the $j$-th process at time $n$, $x_{j,n}$, is expressed by

$$f_{i,j}(x_{i,n+\tau}, x_{j,n}) = \frac{1}{2\pi} \det \left[ \begin{array}{cc} 1 & r_{ij,\tau} \\ r_{ij,\tau} & 1 \end{array} \right]^{-1/2} \det \left[ \begin{array}{c} x_{i,n+\tau} \\ x_{j,n} \end{array} \right] \times \exp \left( -\frac{1}{2} \begin{array}{c} x_{i,n+\tau} \\ x_{j,n} \end{array} \left[ \begin{array}{cc} 1 & r_{ij,\tau} \\ r_{ij,\tau} & 1 \end{array} \right] \begin{array}{c} x_{i,n+\tau} \\ x_{j,n} \end{array} \right) .$$

Substituting $y_{i,n+\tau} = F_{i}^{-1}(\Phi(x_{i,n+\tau}))$ and $y_{j,n} = F_{j}^{-1}(\Phi(x_{j,n}))$ in (7), we obtain

$$f_{i,j}(y_{i,n+\tau}, y_{j,n}) = \frac{1}{2\pi} \times \frac{1}{\det \left[ \begin{array}{cc} 1 & r_{ij,\tau} \\ r_{ij,\tau} & 1 \end{array} \right]^{1/2}} \exp \left( -\frac{1}{2} \begin{array}{c} \Phi^{-1}(F_{i}(y_{i,n+\tau})) \\ \Phi^{-1}(F_{j}(y_{j,n})) \end{array} \left[ \begin{array}{cc} 1 & r_{ij,\tau} \\ r_{ij,\tau} & 1 \end{array} \right] \begin{array}{c} \Phi^{-1}(F_{i}(y_{i,n+\tau})) \\ \Phi^{-1}(F_{j}(y_{j,n})) \end{array} \right) .$$

Denoting $Y = [y_{i,n+\tau} y_{j,n}]^T$ and $X = [x_{i,n+\tau} x_{j,n}]^T$, by Jacobian transformation, we obtain

$$f_{i,j}(y_{i,n+\tau}, y_{j,n}) = \left. \frac{f_{i,j}(x_{i,n+\tau}, x_{j,n})}{|J(Y,X)|} \right|_{x_{i,n+\tau} \rightarrow y_{i,n+\tau}}^{x_{j,n} \rightarrow y_{j,n}} ,$$

where the Jacobian $|J(Y,X)|$ is expressed as

$$|J(Y,X)| = \left. \begin{array}{c} \frac{\partial y_{i,n+\tau}}{\partial x_{i,n+\tau}} \\ \frac{\partial y_{j,n}}{\partial x_{j,n}} \\ \frac{\partial y_{i,n}}{\partial x_{i,n}} \end{array} \right|.$$

By the calculus identity, we obtain

$$\frac{\partial F_{i}^{-1}(\Phi(x_{i,n+\tau}))}{\partial x_{i,n+\tau}} = \left. \frac{\partial F_{i}^{-1}(\Phi(x_{i,n+\tau}))}{\partial \Phi(x_{i,n+\tau})} \right|_{\Phi(x_{i,n+\tau})} \frac{\partial \Phi(x_{i,n+\tau})}{\partial x_{i,n+\tau}} = -\frac{1}{f_{i}(F_{i}^{-1}(\Phi(x_{i,n+\tau})))} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{x_{i,n+\tau}^2}{2} \right) .$$

By the same procedure as (11), we obtain

$$\frac{\partial F_{j}^{-1}(\Phi(x_{j,n}))}{\partial x_{j,n}} = -\frac{x_{j,n}}{\sqrt{2\pi} f_{j}(F_{j}^{-1}(\Phi(x_{j,n})))} .$$

Substituting (11) and (12) into (10), we obtain

$$|J(Y,X)|_{x_{i,n+\tau} \rightarrow y_{i,n+\tau}}^{x_{j,n} \rightarrow y_{j,n}} = \left. \frac{-x_{i,n+\tau}}{f_{i}(F_{i}^{-1}(\Phi(x_{i,n+\tau})))} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{x_{i,n+\tau}^2}{2} \right) \times \exp \left( -\frac{-x_{j,n}}{f_{j}(F_{j}^{-1}(\Phi(x_{j,n})))} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{x_{j,n}^2}{2} \right) \right) \right|_{x_{i,n+\tau} \rightarrow y_{i,n+\tau}}^{x_{j,n} \rightarrow y_{j,n}} \left( \Phi^{-1}(F_{j}(y_{j,n})) \right) = \Phi^{-1}(F_{j}(y_{j,n})) \left( \frac{1}{2\pi} \right)^{1/2} \exp \left( -\frac{1}{2} \left( \Phi^{-1}(F_{j}(y_{j,n})) \right)^T \left( \begin{array}{cc} 1 & r_{ij,\tau} \\ r_{ij,\tau} & 1 \end{array} \right) \left( \Phi^{-1}(F_{j}(y_{j,n})) \right) \right) .$$

Denoting the variance of the $i$-th fading envelope process, $y_{i,n}$, as $\sigma_{i}^2$, by the definition of correlation coefficients, we have

$$\rho_{ij,\tau} = \frac{1}{\sigma_{i}\sigma_{j}} \int_{0}^{\infty} \int_{0}^{\infty} (y_{i,n+\tau} - \mu_{i})(y_{j,n} - \mu_{j}) \times f_{i,j}(y_{i,n+\tau}, y_{j,n}) dy_{i,n+\tau} dy_{j,n} .$$

Substituting (14) into (15), we obtain

$$\rho_{ij,\tau} = \frac{1}{\sigma_{i}\sigma_{j}} \int_{0}^{\infty} \int_{0}^{\infty} (y_{i,n+\tau} - \mu_{i})(y_{j,n} - \mu_{j}) \times f_{i,j}(y_{i,n+\tau}, y_{j,n}) dy_{i,n+\tau} dy_{j,n} \left( \Phi^{-1}(F_{j}(y_{j,n})) \right) = \left. \frac{1}{1/2} \left( \Phi^{-1}(F_{j}(y_{j,n})) \right) \Phi^{-1}(F_{j}(y_{j,n})) \right|_{x_{i,n+\tau} \rightarrow y_{i,n+\tau}}^{x_{j,n} \rightarrow y_{j,n}} \left( \begin{array}{c} \Phi^{-1}(F_{j}(y_{j,n})) \\ \Phi^{-1}(F_{j}(y_{j,n})) \end{array} \right) \left( \begin{array}{c} 1 \\ 1 \end{array} \right) \left( \begin{array}{c} \Phi^{-1}(F_{j}(y_{j,n})) \\ \Phi^{-1}(F_{j}(y_{j,n})) \end{array} \right) \left( \begin{array}{c} 1 \\ 1 \end{array} \right) \times dy_{i,n+\tau} dy_{j,n} .$$

Given explicit values of $\rho_{ij,\tau}$, $f_{i,\tau}$, $F_{i,\tau}$, and other relevant parameters, various iterative numerical methods can be used to solve for $r_{ij,\tau}$ by using (16). Our own experiences shows
accurate numerical solutions can be obtained readily as shown in the following three examples. In the above derivations, the \(i\) and \(j\) are allowed to be equal. Therefore, (16) includes both ACFs, i.e., \(i = j\), and CCFs, i.e., \(i \neq j\). It is noted that the \(\rho_{ij,\tau}\) and \(\rho_{ij,\tau}\) are equal to 1 for \(\forall (i, j, \tau) \in \{i, j, \tau | (i = j) \cap (\tau = 0)\}\). Thus, we only need to evaluate \(\rho_{ij,\tau}\) from \(\rho_{ij,\tau}\) using (16) for \(\forall (i, j, \tau) \notin \{i, j, \tau | (i = j) \cap (\tau = 0)\}\).

C. Summary of the Proposed Approach

For the given \(\rho_{ij,\tau}\), \(F_i(y_i)\), and \(f_i(y_i)\), perform the following: 1) Numerically evaluate the \(\rho_{ij,\tau}\) corresponding to the given \(\rho_{ij,\tau}\), \(F_i(y_i)\), and \(f_i(y_i)\) using (16); 2) Evaluate the parameters, \(A_i\) and \(Q_i\), of the Gaussian vector AR by (5) and (6); 3) Generate samples, \(x_{i,n}\), from the Gaussian vector AR from \(A_i\) and \(Q_i\); 4) Perform \(y_{i,n} = F_i^{-1}(\Phi(x_{i,n}))\). The generated \(y_{i,n}\) have the desired processes with the desired \(\rho_{ij,\tau}\), \(F_i(y_i)\), and \(f_i(y_i)\).

III. PROPERTIES AND DISCUSSIONS

In the design of the proposed approach, we used \(\rho_{ij,\tau} = 1\) for \(\forall (i, j, \tau) \in \{i, j, \tau | (i = j) \cap (\tau = 0)\}\) in the Yule-Walker equations. The variances, i.e., \(r_{ii,0}^2 \forall i\), of the generated individual processes in the Gaussian vector AR process are all equal to 1. In other words, the marginal pdfs of the processes in the Gaussian vector AR process generated by this approach are normalized Gaussian pdfs. Therefore, for the normalized Gaussian cdf \(\Phi(\cdot)\) and normalized Gaussian \(x_{i,n}\), the inverse transform sampling theorem [22] states that the \(\Phi(x_{i,n})\) is uniformly distributed. Since \(\Phi(x_{i,n})\) is uniform distribution, by the inverse transform sampling theorem [22], the \(y_{i,n} = F_i^{-1}(\Phi(x_{i,n}))\) is the random variable distributed by the cdf \(F_i(.)\), with its pdf \(f_i(.)\). The choices of \(F_i(.)\) in the inverse transform sampling, i.e., \(y_{i,n} = F_i^{-1}(\Phi(x_{i,n}))\) in Fig. 1., allow the output pdfs to be heterogeneous. Therefore, by choosing various \(F_i(.)\) for different \(i\), our proposed approach is able to generate correlated fading envelope processes of heterogeneous pdfs.

The solution of Gaussian vector AR parameters in (5), \(A = R_{xx}^{-1}V\), sometimes produces unstable Gaussian vector AR or the unrealizable \(Q\) due to the ill-conditioned \(R_{xx}\) matrix. To avoid this problem, we adopt a regularization approach [2][3], i.e., using \(A = (R_{xx} + \varepsilon I)^{-1}V\), where \(\varepsilon\) is a small number and \(I\) is the identity matrix. The choices of \(\varepsilon\) are considered in [3].

IV. SIMULATIONS

Three examples are demonstrated in this section. The first example is a scenario with a single auto-correlated Nakagami channel [19]. The second example is a 2x2 MIMO scenario, where 4 Rayleigh channels are generated [2]. The parameters and simulation settings of the above two examples are selected from previously published results in [2] and [19]. The third example is a scenario with 3 channels of heterogeneous pdfs, i.e., the 3 channels with Nakagami, Rician, and Rayleigh pdfs individually. The parameters of the third example are selected to demonstrate the capability of generating correlated heterogeneous channel envelopes.
A. Example 1: The Single Nakagami Channel

In this example, the settings and parameters of this example are selected from [19]. The Nakagami pdf and cdf are respectively expressed as

\[ f_{Naka}(y) = \frac{2m^m}{\Gamma(m)\omega^m} y^{2m-1} \exp\left(-\frac{m}{\omega} y^2\right), \tag{17} \]

\[ F_{Naka}(y) = \Re\left\{ m, \frac{m}{\omega} y^2 \right\}, \tag{18} \]

where \( \Gamma(\cdot) \) is the Gamma function and \( \Re\{\cdot\} \) is the regularized incomplete Gamma function. The mean of the Nakagami random variable described by (17) is \( \frac{\Gamma(m+1/2)}{\Gamma(m)} \left( \frac{m}{\omega} \right)^{1/2} \). The parameters are chosen as \( m = 1.69 \) and \( \omega = 1.16 \) so that the Nakagami process has the mean of 1. The theoretical, i.e., the desired, ACF is chosen as \( \rho_{Naka,\tau} = J_0^2(2\pi f_D \Delta|\tau|) \), where the \( \tau \) represents the index of the time lag, the \( \Delta \) represents the sampling interval, the Doppler shift is \( f_D = \frac{v}{\lambda} = \frac{v f_c}{3 \times 10^8} \) Hz, \( v = 27.78 \) m/s. There are two settings for the sampling rates, i.e., the low sampling rate and the high sampling rate.

In the low sampling rate scenario, the carrier frequency is \( f_c = 1.8 \) GHz and the sampling rate is 9,600 Hz, which corresponds to \( \Delta = 1/9600 \) second. In the high sampling rate scenario, the carrier frequency is \( f_c = 900 \) MHz and the sampling rate is 24,300 Hz, which corresponds to \( \Delta = 1/24300 \) second.

The empirical and theoretical ACFs are shown in Fig. 2. Comparing the empirical ACFs with the theoretical ACFs, it is observed that the empirical ACFs closely follow the theoretical ACFs. The empirical cdfs of the generated fading envelope samples and the corresponding theoretical cdfs are compared in Fig. 3, where the empirical cdfs closely follow the theoretical cdfs except at the tails. To further verify the empirical cdfs, we conducted Kolmogorov-Smirnov [30] tests (KSTs) on the generated fading envelopes with their respective theoretical cdfs. The resulting p-values of the KSTs are 0.104 and 0.375, which pass the KSTs with significance level of 0.05, respectively for the low sampling rate and high sampling rate scenarios. The observations of the cdfs in Fig. 3 and the KST results indicate the achievement of the design goal that the targeted distribution is the Nakagami distribution. The observations in Fig. 2 and 3 and the KST results verify the capabilities of the proposed approach to generate the desired auto-correlated Nakagami fading envelopes.

B. Example 2: The 2x2 Rayleigh MIMO channels

In this 2x2 Rayleigh MIMO example, there are a total of four Rayleigh channels, with parameters selected from [2]. The Rayleigh pdf and cdf are expressed as

\[ f_{Ray}(y) = \frac{y \exp\left(-\frac{y^2}{\sigma_{Ray}^2}\right)}{\sigma_{Ray}}, \tag{19} \]

\[ F_{Ray}(y) = 1 - \exp\left(-\frac{y^2}{2\sigma_{Ray}^2}\right), \tag{20} \]

The parameter is chosen as \( \sigma_{Ray} = 0.7079 \) for all the four envelope processes such that the four envelope processes have their means equal to 1. The ACFs and CCFs of the four channels are chosen as

\[ \rho_{11,\tau} = \rho_{22,\tau} = \rho_{33,\tau} = \rho_{44,\tau} = J_0(2\pi f_D \Delta|\tau|), \]

\[ \rho_{12,\tau} = \rho_{34,\tau} = J_0\left(\{a^2 + b^2 - 2ab \cos(\beta - \gamma)\}^{1/2}\right), \]

\[ \rho_{13,\tau} = \rho_{24,\tau} = J_0\left(\{a^2 + c^2 - 2ac \sin(\alpha) \sin(\gamma)\}^{1/2}\right), \]

\[ \rho_{14,\tau} = \rho_{23,\tau} = J_0\left(\{a^2 + b^2 + c^2 - 2ab \cos(\beta - \gamma) - 2ac \sin(\alpha) \sin(\gamma) - b \sin(\beta)\}\right)^{1/2}, \]

where \( f_D \Delta \) is the maximum Doppler frequency shift normalized by the sampling time \( \Delta \), and the \( \tau \) represents the index of the time lag. The parameters are \( a = 2\pi f_D \Delta|\tau|, b = 2\pi d/\lambda, \) and \( c = 2\pi f_D \Delta|\tau| \).
c = 2\pi\delta/\lambda. The geometric interpretations of these parameters in the propagation environment are detailed in [2] and [4]. The values of the parameters are chosen as \( fD_\Delta = 0.045 \), d/\lambda = 0.5, \delta/\lambda = 17, \eta = 4, \alpha = \beta = 90, \) and \( \gamma = 0 \) [2]. The empirical and theoretical ACFs and CCFs are shown in Fig. 4 and 5. Comparing the empirical ACFs and CCFs with their theoretical counterparts, it is observed that the empirical ACFs and CCFs closely follow their theoretical counterparts. The empirical cdfs and the theoretical cdfs are compared in Fig. 6, where the empirical cdfs closely follow the theoretical cdfs. Besides observing the empirical and theoretical cdfs, we conducted KSTs on the generated fading envelopes with their respective theoretical cdfs. The resulting p-values of the KSTs are 0.057, 0.49, 0.32, and 0.098 respectively for the four Rayleigh channels, which pass the KSTs with significance level of 0.05. The observations of the cdfs in Fig. 6 and the KST results verify the effectiveness of the proposed approach in generating the desired multiple fading envelopes of Rayleigh pdfs.

C. Example 3: The Three Envelope Processes with Nakagami, Rician, and Rayleigh pdfs

In this example, there are three envelope processes each having the Nakagami, Rician, and Rayleigh pdfs respectively. This scenario is designed to demonstrate the capability of the proposed approach to generate envelope processes of different pdfs. The Rician pdf and cdf are expressed as

\[
    f_{\text{Rician}}(y) = \frac{y}{\sigma_{Ri}^3} \exp\left(-\frac{y^2 + v_{Ri}^2}{2\sigma_{Ri}^2}\right) I_0\left(\frac{yv_{Ri}}{\sigma_{Ri}^2}\right),
\]

\[
    F_{\text{Rician}}(y) = 1 - Q\left(\frac{v_{Ri}}{\sigma_{Ri}}, \frac{y}{\sigma_{Ri}}\right),
\]

where \( I_0(\cdot) \) is the modified Bessel function of the first kind with order zero and \( Q(\cdot, \cdot) \) is the Marcum Q-function. In the settings of this example, the first envelope process is the Nakagami process with its pdf (17) parameterized by \( m = 2.5 \) and \( \omega = 1 \), such that its mean is equal to 0.95 and its variance is equal to 0.095. The second envelope process is the Rician process with its pdf (25) parameterized by \( \sigma_{Ri} = 1 \) and \( v_{Ri} = 1 \), such that its mean is equal to 1.55 and its variance is equal to 0.602. The third envelope process is the Rayleigh process with its pdf (19) parameterized by \( \sigma_{Ray} = 1 \), such that its mean is equal to 1.25 and its variance is equal to 0.43. The settings and parameters are intentionally chosen to create 3 fading envelope processes with different means, variances,
and pdfs. The ACFs and CCFs are chosen as the exponential type [25][26][27], expressed as

\[
\rho_{11,\tau} = \rho_{22,\tau} = \rho_{33,\tau} = \exp(-f_D \Delta |\tau|), \quad \rho_{12,\tau} = \rho_{21,\tau} = \rho_{13,\tau} = \rho_{31,\tau} = \rho_{23,\tau} = \rho_{32,\tau} = \kappa \exp(-f_D \Delta |\tau|),
\]

where \( \tau \) represents the index of the time lag and the \( f_D \Delta \) is the decay constant of the correlations. The parameters are chosen as \( f_D \Delta = 0.4 \) and \( \kappa = 0.6 \). The generated and theoretical ACFs and CCFs are shown in Fig. 7 and 8. Comparing the empirical ACFs and CCFs with their theoretical counterparts, it is observed that the empirical ACFs and CCFs closely follow the theoretical counterparts. The empirical cdfs and the theoretical cdfs are compared in Fig. 9. The empirical cdfs are observed to closely follow the theoretical cdfs. Besides observing the empirical and theoretical cdfs, we conducted KSTs on the generated envelopes with their respective theoretical cdfs. The resulting p-values of the KSTs are 0.086, 0.073, and 0.23, respectively for the Nakagami, Rician, and Rayleigh channels, which pass the KSTs with significance level of 0.05. The observations of the cdfs in Fig. 9 and the KST results show that the generated fading envelopes of the three channels closely follow Nakagami, Rician, and Rayleigh distributions respectively. The observations in Fig. 7-9 and the KST results demonstrate the capabilities of the proposed approach to generate the desired auto-correlated and cross-correlated heterogeneous fading envelope processes. In Fig. 10, the empirical and theoretical un-normalized ACFs are shown to close to each other.

V. Conclusions

The previous research efforts have focused on generating correlated fading processes of the same family. The proposed unified approach uses Gaussian vector AR process modeling and the inverse transform sampling method. We derive the equations to compute the required correlations of the Gaussian vector AR process from the intended ACFs, CCFs, and pdfs of the fading envelope processes. By controlling the correlations of the Gaussian vector AR process from the Yule-Walker equation, the output processes are driven to meet the desired ACFs, CCFs, and pdfs.

Since the proposed approach is more general, our approach includes generating the correlated processes of the same family as the special cases. In the first example, the single Nakagami process is generated. In the second example, the multiple Rayleigh processes are generated. In the third example, to demonstrate the capability to generate correlated fading envelope processes of heterogeneous pdfs, we demonstrate the generation of three correlated envelope processes each having Nakagami, Rician, and Rayleigh pdfs respectively. The sample ACFs and CCFs are observed to closely follow their theoretical counterparts. The generation of the fading processes with the desired properties, i.e., the ACFs, CCFs, and pdfs, is crucial in evaluating the performances of various diversity systems. The proposed approach is capable of generating fading envelope processes with heterogeneous pdfs and desired correlations. The numerical accuracies of the proposed approach appear to be acceptable as observed in the examples.

Acknowledgments

The authors would like to thank the anonymous reviewers for their valuable comments. Kung Yao also wishes to thank the Royal Society Kan Tong Po Visiting Professorship grant at the HK Polytechnic University where parts of this manuscript were prepared.

References


